# Stability Experiment of a Biped Walking Robot with Inverted Pendulum 

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#### Abstract

This paper is concerned with a balancing motion formulation and control of the ZMP (Zero Moment Point) for a biped walking robot that has a balancing weight of an inverted pendulum. The original dynamic stability equation of a walking robot is nonlinear; because it's balancing weight is an inverted pendulum type. The stablization equation of a biped walking robot is modeled as a linearized non-homogeneous second order differential equation with boundary conditions. With the FDM (Finite Difference Method) solution of the linearized differential equation, a trajectory of balancing weight can be directly calculated. Furthermore, it makes that possible input the desired ZMP or ZMP trajectory for various gait, situation and complex motion. Also, it can be easily approximated that a balancing range or motion when some link parameters are changed; especially a balancing mass is changed. In this paper, the simulator with a balancing weight of an inverted pendulum is programmed to get and calculate the desired ZMP and the actual ZMP. The operating program is developed for a real biped walking robot IWRII-IP. Walking of 6 steps will be simulated and experimented with a real biped walking robot. This balancing system will be applied to a biped humanoid robot, which consist legs and upper body, at future work.


## I. Introduction

Many studies have been performed concerning a biped walking robot for over 30 years. These studies are classified into two main branches according to their aims, the control of a walking motion of a biped walking robot [1-2] and the stabilization of a biped walking robot [3-4]. The former is concerned with the control of dynamic properties in dynamic walking, gait control using force or torque sensor, stability analysis in state space, and so on [5-6]. The latter is concerned with the stability analysis using the ZMP concept, gait control and walking analysis under an external force, system stabilization with a balancing joint, and so on [7-9]. The latter class is very important in a realization of biped walking. But the major problem associated with analysis and control of stability on a biped walking robot is difficult to predict and to verify the system stability.

The proper biped locomotion, which was used as the simulation model or realized, can be put into two classifications according to its structure of robot. In former days, a biped locomotion was usually made up of only lower limbs. Recently, some biped locomotion has a balancing joint with a balancing weight that compensates the system stability. In the case of walking in different environments, a biped walking robot must have a capability to change its gait like walking periods, turning, strides and so on [10-13]. A biped walking robot that has only the lower limbs, however, does not have a capability to change its gait, or due to its needs having a much more complicated algorithm. It is also difficult to verify the

ZMP trajectory or to define the balancing motion before it walks.

To have more flexible walking locomotion [14-15], consideration of a balancing joint began. Its purpose was the control ZMP or the stabilizing of biped locomotion. There are two kinds of balancing joints in a biped locomotion, one is made up of a prismatic joint and another is made up of revolute joints for compensating dynamic properties of swing leg and its gravitational acceleration. In the case of a prismatic joint balancing, it usually consists of a roll joint and a prismatic joint. It does not have any upward (or downward) directional acceleration which is generated by balancing joints. Therefore, it is easy to make some linearized mathematical model. In the case of a revolute joint, it usually consists a roll joint with a pitch joint for balancing motion [16-17]. Those revolute joints make some polar motion of a balancing weight that can be represented by Cartesian space parameter with highly coupled term.

In this paper, the linearized mathematical model with a balancing weight that is inverted pendulum type is introduced. To get the dynamically stable walking, the trajectories of balancing joints are solved by the FDM with boundary conditions. After that, the stability of biped walking robot is verified by the ZMP equation.

## II. Mathematical Model

For a dynamics analysis, let each vector be defined as shown in Fig. 1 on a reference coordinate frame. " $i$ " is an index of an each link, $m_{i}$ is the mass of a particle $i$ and $\bar{r}_{i}$ is the position vector of a particle i. $\vec{P}_{i}$ is the position vector of a point $P$ on the $X Y$ plane.


Fig. 1 Vector representation
By applying D'Alembert principle, the equation of motion at an arbitrary point $P$ is given by

$$
\begin{equation*}
\sum_{i=0}^{8} m_{i}\left(\vec{r}_{i}-\vec{P}\right) \times\left(\overrightarrow{\vec{r}}_{i}+\vec{G}\right)+\vec{M}_{T}=0 \tag{1}
\end{equation*}
$$

where $\overline{\vec{r}}_{i}$ is the acceleration vector of a particle i and $\vec{G}_{i}$ is the gravitational constant acceleration vector.
$\bar{M}_{i}$ is the total moment vector that is acted on a point P . By rearranging (1), the information of the actual ZMP can be obtained as (2). Applying $\vec{P}_{t}$ with ZMP and arranging about $\mathrm{ZMP},(2)$ can be driven. $\mathrm{X}_{\text {zпр }}$ and $\mathrm{Y}_{\text {zup }}$ are ZMP about X -axis and Y -axis. $x_{i}, y_{i}$ and $z_{i}$ are positions of a particle i , and $\ddot{x}_{i}, \ddot{y}_{i}$ and $\ddot{z}_{i}$ are accelerations of a particle i. $\mathrm{G}_{\mathrm{x}}, \mathrm{G}_{y}$ and $\mathrm{G}_{\mathrm{z}}$ are the gravitational constant accelerations about X -axis, Y-axis and Z-axis. A stable ZMP should be inside a supporting area that circumscribes one or two soles in support.


If trajectories of both legs and the desired ZMP are given, the motion of balancing weight that stabilize the walking of a robot is determined by two balancing equations for sagittal and lateral direction. They can be obtained by arranging equation (2) for the balancing weight $m_{0}$, which are shown as follows.

$$
\begin{align*}
& \bar{x}_{0} \frac{\left(\tilde{z}_{0}+G_{)}\right)}{z_{0}} x_{0}=\alpha \\
& \bar{y}_{0} \frac{\left(\bar{z}_{0}+G_{2}\right)}{z_{2}} y_{0}=\beta \tag{3}
\end{align*}
$$

where $\bar{x}_{0}, \bar{y}_{0}$ and $\ddot{z}_{0}$ are acceleration values of particle $m_{0}$. $x_{0}$, $y_{0}$ and $z_{0}$ are position values of particle $m_{0}$. (3) represents a dynamic property of a balancing weight. By the factor of a inverted pendulum type balancing weight, (3) have a non-linearity, $\alpha$ and $\beta$ are given by

$$
\begin{align*}
& \gamma=\frac{\left(\tilde{z}_{0}+G_{2}\right)}{z_{0}} \tag{4}
\end{align*}
$$

where $x^{*}{ }_{\mathrm{mpp}}$ and $y^{*}{ }_{\mathrm{m} / \mathrm{y}}$ are the points or trajectories of the desired ZMP on the XY plane. (4) can be simplified into (5). It contains a highly coupled term $\gamma_{0}(t)$ that is caused by revolute joints for balancing motion. To get the motion of a balancing weight $\gamma_{0}(t)$ is linearized to constant with boundary conditions.

$$
\begin{align*}
& \ddot{x}_{0}(t)-\gamma_{0}(t) x_{0}(t)=\alpha(t) \\
& \ddot{y}_{0}(t)-\gamma_{0}(t) y_{0}(t)=\beta(t) \tag{5}
\end{align*}
$$

## III. System

The system consists of a real biped walking robot IWRIII-IP and a control system. A biped walking robot has eight AC servomotors and reducers, and DSP controller embedded in the host computer that analyzes and monitors the overall robot system controls a biped robot. Fig. 2 represents the total system structure of a biped walking robot.


Fig. 2 Total system of a biped walking robot
The host computer includes the simulator that plans reference trajectories of leg and the desired ZMP through user define constraints. The simulator generates trajectories of balancing weight by the FDM. Also a host computer includes an operating program coded by Visual $\mathrm{C}++$.

The controller, which uses TMS320C31 DSP as a main CPU and a dual port memory for a high-speed communication, is embedded in a PC environment. The PID \& feed forward control scheme using memory buffer for 8 servos is done for every 10 ms in a DSP controller, and encoder feedback datum are acquired for every 50 ms .

Fig. 3 represents mass models of a biped walking robot with nine rigid bodies and a coordinate system. A mass model is changed at change of support leg. An origin frame of a robot locates to a center of a hip.


Fig. 3 Mass models and a coordinate system

A real biped walking robot has three degrees of freedom in each legs and two degrees of freedom in balancing joints that include a roll joint and a pitch joint. 8 AC servo motors and reducers are included in a robot. AC servo motors consist of 400W for knees and 200W for the others. Gear ratios are 100:1 at each ankle joints and 60:1 at the others.

The total mass of a biped walking robot is about 52.96 Kg . Link length between ankle and lnee and between knee and hip is 0.15 m . Link length from sole to ankJe is 0.0885 m and length from hip to balancing yaw joint is 0.046 m . Length of balancing weight is 0.215 m and total length is 0.6645 m .

Table 1. represents each mass and centroid of link at each support leg. This data is used at calculating the acceleration of mass by Newton-Euler recursive method.

|  | TABLE 1. MASS AND CENTROD |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | Centroid about the local frame |  |  |  |  |  |
|  |  | Left Leg Support |  |  | Right Leg Support |  |  |
| M0 | 19.00 | 0.000 | 0.252 | 0.000 | 0.000 | 0.252 | 0.000 |
| M1 | 2.11 | 0.015 | -0.009 | 0.043 | 0.015 | 0.009 | 0.043 |
| M2 | 3.77 | 0.036 | 0.000 | 0.040 | 0.036 | 0.000 | -0.040 |
| M3 | 6.23 | 0.070 | 0.000 | 0.092 | .070 | 0.000 | -0.092 |
| M4 | 5.14 | 0.050 | 0.000 | -0.046 | 0.050 | 0.000 | 0.046 |
| M5 | 6.23 | 0.080 | 0.000 | -0.092 | 0.080 | 0.000 | 0.092 |
| M6 | 3.77 | 0.114 | 0.000 | -0.040 | 0.114 | 0.000 | 0.040 |
| M7 | 2.11 | 0.045 | -0.015 | 0.009 | 0.045 | -0.015 | -0.009 |
| M8 | 4.60 | 0.000 | 0.058 | -0.023 | 0.000 | 0.058 | -0.023 |

## IV. Simulation

The simulator includes 3-Dimensioal graphic user interface, language module of user define file, kinematics module, dynamic module, CAD interface module, ODE solver module and walking stability module. It is programmed with Visual $\mathrm{C}++$. Fig. 4 is the structure of a simulator.


Fig. 4 Simulator structure
Fig. 5 shows the user-defined file of a first step. The step size is 0.28 m . During 2 seconds, right leg moves up forward and the center of a robot moves forward half size of a stride in

2 seconds. After that, the robot changes a support phase during 1 second. The trajectory of desired ZMP is a simple straight line that is inside foot of robot selects. User can optimize that trajectory to a 5 th spline curve by change via point's data.


Fig. 5 Example of unit step file
The 3-Dimensioal graphic simulator interprets the walking file that describes step parameters. It generates joint data, Cartesian data of foot, the desired ZMP and the actual ZMP. An operator can easily and intuitively implement input and output data through the simulator. Fig. 6 shows the picture of a 3-Dimensioal simulator.


Fig. 6 Picture of the simulator with 3-Demensioal graphic
Fig. 7 represents trajectories of joint position by a simulator. Total steps are 6 steps and total displacement is 1.40 m . Total time is 18 seconds. According to Fig. 7, continuity of each joint's trajectories is guaranteed at junctions of every step. These trajectories are used to verify the actual ZMP and to get some dynamic properties.


Fig. 7 Trajectories of joint position

Fig. 8 is the comparison of an input desired ZMP and an output actual ZMP. (a) is a first step of 6 steps, (b) is a second and forth step of 6 steps. (c) is a third and fifth step of 6 steps. (d) is a final step. During a single leg support time, the error between desired ZMP and actual ZMP is limited to 0.01 m . But the error increase in dual leg support time. The reason is mechanical constrains that requires avoidance of singular case.


Fig. 8 Trajectories of the desire ZMP and the actual ZMP

The trajectory of a balancing weight is reconsructed in joint space for an above mechanical constraint, and this causes some error between a desired ZMP and an actual ZMP. A biped walking robot with balancing weight of inverted pendulum can dynamic stable walk according simulation result.

## V. Experiment

Experiment with a real biped walking robot IWRIII-IP according to simulation data is executed. Fig. 9 is a picture of initial pusture of experiment.


Fig. 9 Picture of a real biped walking robot INRII-IP
Fig. 10 is the operating program of the robot. This program is operated in a host computer that embed a motion controller. An operating system of a host computer is Windows 9 SE . It has an interface of motion controller and a file interface. Operator can move each joint or can change the PID parameter of a motion controller. A simulation data can be inputted to an operating program.


Fig. 10 Picture of an operating program

Fig. 11 represents a result of an experiment. The sampling time is 0.05 second. It shows the joint position errors of right, left legs and balancing joints during a walking. According to (a) ~(f), errors of legs are limited to 250 pulses. (g) and (h) are errors of balancing joints, errors of balancing pitch reach to 900 pulses in maximum. It is caused to the gravity effect of an inverted pendulum type balancing weight. It is shown that these errors have little effect on the robot links positions.


Fig. 11 Experimental data of position error

## VI. Conclusion and Future Work

A balancing motion formulation and control of ZMP for a biped walking robot that has a balancing weight of inverted pendulum type are simulated and experimented to a real biped walking robot. An inverted pendulum is modeled as a linearized differential equation with boundary condition. The trajectory of balancing weight can be solved by FDM. The actual ZMP verify the stability of walking. By the experiment, a linearized model is available for a biped walking robot with an inverted pendulum. The difference between a desired ZMP and an actual ZMP requires a compensation algorithm for a linearized model.

This result will be applied and expanded to a biped humanoid robot that has arms, vision system and FSR (Force Sensing Resistor) at future work.

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