Humanoid Walking Robot Control with Natural ZMP References

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***Abstract*— Humanoid robotics attracted the attention of many researchers in the past 35 years. The motivation of research is the suitability of the biped structure for tasks in the human environment. The control of a biped humanoid is a challenging task due to the hard-to-stabilize dynamics.**

Walking reference trajectory generation is one of the key problems. The Zero Moment Point (ZMP) Criterion is widely employed in the stability analysis of biped robot walk. Linear Inverted Pendulum Model (LIPM) based reference generation obtained by applying the ZMP Criterion are reported in the literature. In these methods, generally, the ZMP during a stepping motion is kept fixed in the middle of the supporting foot sole.

This paper employs a reference generation algorithm based on the Linear Inverted Pendulum Model (LIPM) and moving support foot ZMP references. This kind of reference generation possesses naturalness, in that, the ZMP in the human walk does not stay fixed, but it moves forward, under the supporting foot. The application of Fourier series approximation simplifies the solution and it generates a smooth ZMP reference. A suitable swing foot reference is employed too. A number of modes of trajectory and force control methods for locomotion are devised and applied. The developed techniques are tested through simulation with a 12 DOF biped robot model. The results obtained are promising for implementations.

I. INTRODUCTION

Humanoid robotics attracted the attention of many researchers in the past 3 decades. It is currently one of the most exciting topics in the field of robotics and there are many projects all around the world [1-6]. The motivation of research is the suitability of the biped structure for tasks in the human environment.

The control of a biped humanoid is a challenging task due to the many degrees of freedom involved and the non-linear and hard to stabilize dynamics [7-8].

Walking reference trajectory generation is a key problem. Methods ranging from trial and error to the use of optimization techniques with energy or control effort minimization constraints are applied as solutions.

A very intuitive criterion used for the reference generation is that the reference trajectory should be suitable to be followed by the robot with its natural dynamics, without the use of extensive control intervention. Reference generation techniques with the so-called Linear Inverted Pendulum Model are based on this idea [9-11]. Simply stated, the walking cycle is then achieved by letting the robot start falling into the walking direction and to switch supporting legs to avoid the complete falling of the robot.

Yet another intuitive demand for the biped robot reference generation is that the reference trajectory should be a stable one, in the sense that it should not lead to unrecoverable falling motion. The Zero Moment Point Criterion [7] introduced to the robotics literature in early 1970s is widely employed in the stability analysis of biped robot walk. Improved versions of the Linear Inverted Pendulum Model based reference generation, obtained by applying the Zero Moment Point criterion in the design process, are reported too [12]. In this approach the Zero Moment Point during a stepping motion is kept fixed in the middle of the supporting foot sole for the stability, while the robot center of mass is following the Linear Inverted Pendulum path.

Although reference generation with the Linear Inverted Pendulum Model and fixed Zero Moment Point reference positions is the technique employed for the most successful biped robots today, this kind of reference generation lacks naturalness at one point. Investigations revealed that the Zero Moment Point in the human walk does not stay fixed under the supporting foot. Rather, it moves forward from the heel to the toe direction [13-15].

In [15], Zhu et. al propose this idea of using variable ZMP to generate a dynamically stable gait in terms of linear inverted pendulum approach. They consider it to follow first order functions from the heel to toe of the foot in single support phase.

This paper employs a reference generation technique based on the Linear Inverted Pendulum Model and moving support foot Zero Moment Point references [16]. The application of Fourier series approximation to the solutions of the Linear Inverted Pendulum dynamics equations simplifies the solution as in [17], and it generates a smooth Zero Moment Point reference for the double support phase.

Foot reference trajectory generation methods for smooth swing foot trajectories, trajectory control methods for the center of mass of the robot and force control techniques for the landing foot are proposed in this paper too.

The reference generation and control techniques are simulated and animated in a 3-D full dynamics simulation environment with a 12 DOF biped robot model. The results obtained are promising for implementations.

The reference generation with natural moving ZMP trajectories is outlined in Section II. The control for locomotion is discussed in Section III. Section IV presents the simulation results and their analysis. The conclusion is drawn lastly.

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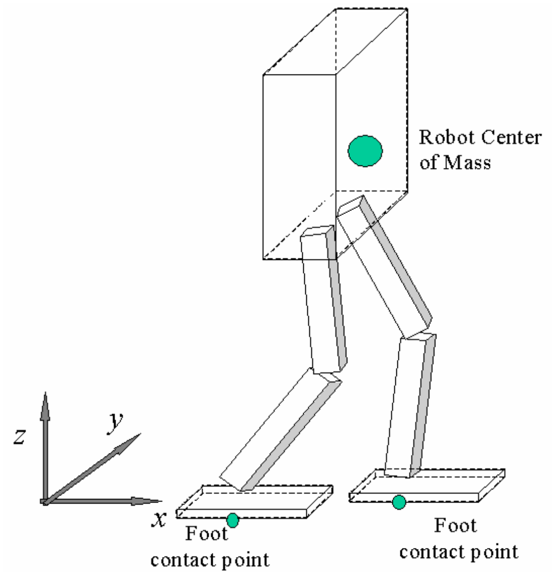


Fig. 1. Biped robot coordinate frames.

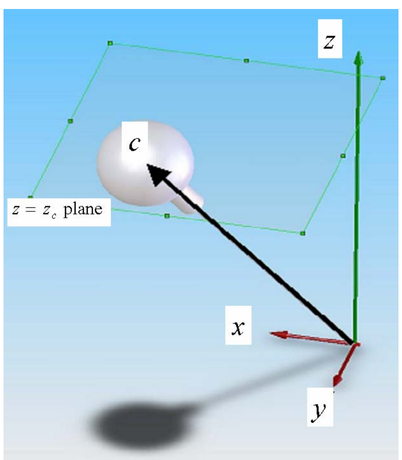


Fig. 2. The Linear Inverted Pendulum.

II. REFERENCE GENERATION WITH NATURAL ZMP TRAJECTORIES

The discussion is mainly independent from the kinematic arrangement of the legged robot. However two assumptions are made. The first one is that the mass of the legs should be much less than the robot mass. In fact, the method is still applicable with heavy legs, but with a degradation of relative stability of the walk. The second assumption is that, the legs should have at least six degrees of freedom (dof). The sketch in Fig. 1 shows a good example of a robot structure for which the reference generation and control algorithms presented below can be applied: A biped robot with six-dof legs.

The full dynamics description for a robot structure like the one shown in Fig. 1 is highly nonlinear, multiple dof and coupled. The closed form solutions of the dynamic equations are difficult to obtain. Rather, Newton-Euler recursive formulations are used in their computation [18-20].

Though obtaining such a model is very useful for the simulation and test of reference generation and control methods, the structure in (1) is too complex to serve as an intuitive model which can help in developing basics and guidelines for the walking control. Simpler models can be more suitable for controller synthesis. The linear inverted pendulum model is such a model. The body (trunk) is approximated by a point mass concentrated at the CoM of the robot. This point mass is linked to a stable (not sliding) contact point on the ground via a massless rod, which is the idealized model of a supporting leg. The swing leg is assumed to be massless too. Fig. 2 shows a linear inverted pendulum. In this figure, $c = (c_x \quad c_y \quad c_z)^T$ stands for the coordinates of this point mass. The height of the CoM is fixed at z_c. This model is called LIPM and it is simple enough to work on and devise algorithms for reference generation[20]. The equations of motion of the CoM with the LIPM are as follows.

$$\ddot{c}_x = \frac{g}{z_c}x + \frac{1}{mz_c}u_p \tag{1}$$

$$\ddot{c}_y = \frac{g}{z_c}y - \frac{1}{mz_c}u_r \tag{2}$$

In this equation m is the mass of the body (point mass), g is the gravity constant (9.806 $\mathrm{m/s^2}$). u_p and u_r are the pitch (about y-axis) and roll (about x-axis) control torques, respectively. These act at the support point (origin in Fig. 2) of the linear inverted pendulum.

Since the solutions of (1) and (2) in the unactuated case ($u_p = u_r = 0$) are unbounded functions, the computation of a reference trajectory from freely falling segments should be carried out carefully in order to obtain a stable reference.

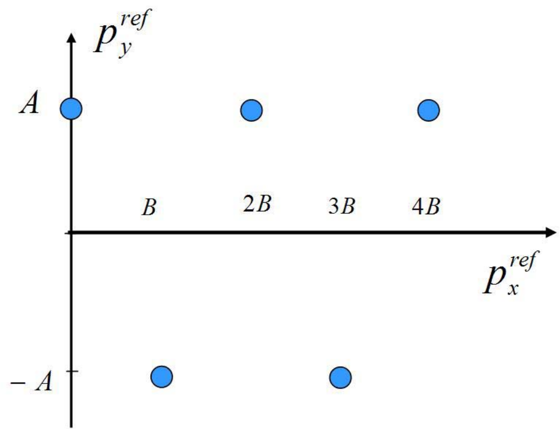
Stability of the walk is the most wanted feature of a reference trajectory. In biped robotics, the most widely accepted criterion for stability is based on the location of the ZMP [7]. For the arrangement in Fig. 2, the zero moment point is defined as the point on the $x-y$ plane about which no horizontal torque components exist. The expressions for the ZMP coordinates p_x and p_y for the point mass structure in Fig. 2 are [17]

$$p_x = c_x - \frac{z_c}{g}\ddot{c}_x \tag{3}$$

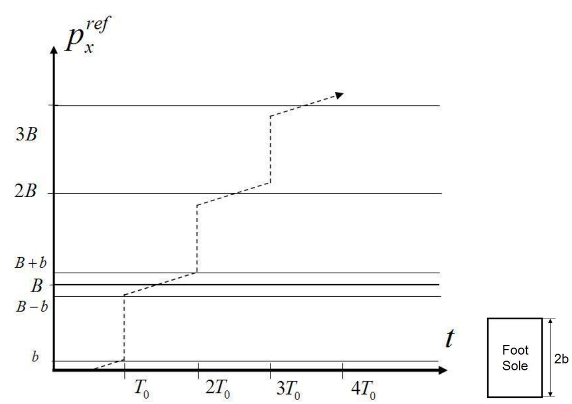
$$p_y = c_y - \frac{z_c}{g}\ddot{c}_y \,. \tag{4}$$

(3) and (4) are equations relating the ZMP and the CoM. For reference generation purposes a suitable ZMP trajectory can be assigned. The only constraint for stability of the robot is that the ZMP should always lie in the supporting polygon defined by the foot or feet touching the ground. The most intuitive choice for the ZMP location is the middle of the

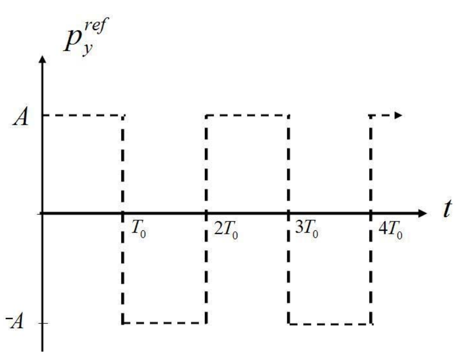
supporting foot sole. However, the naturalness of the walk cannot be addressed by this choice. Investigations of the human ZMP revealed that it moves forward under the sole. Further, the ZMP references in natural human walk do not move from left support foot center to the right support foot center and vice versa instantaneously. The non-instantaneous transition from left single support phase to the right support phase (with a double support phase) requires a smooth ZMP trajectory in the y-direction in Fig.1 (perpendicular to the walking direction x).



a)



b)



c)

Fig. 3. The preplanned ZMP reference trajectory

The ZMP reference trajectory shown in Fig. 3 takes the forward move of the ZMP under the sole into consideration. However, smooth transition between single foot support phases with a double support phase is achieved by an additional smoothing action.

In Fig. 3, A is the half of the distance between the foot centers in the y-direction, B is the step size, b is the half of the foot size (or equivalently the half of the distance traveled by ZMP under the single supporting foot) and T is half of the walk cycle period. As can be observed from Fig. 3, firstly, step locations are determined. The selection of the step locations can be based on the size of the robot and the nature of the task performed by the robot. The inclined staircase-like p_z and the square-wave structured p_y curves are fully defined by the selection of support foot locations if the half period T is given too. Similar to the step size, the step period can also be determined by the physical size and properties of the robot and by the application.

Having defined the curves, and hence the mathematical functions for p_z and p_y, the equations (3), (4) and Laplace transform techniques can be employed for the solution of c_x and c_y. After having found the required c_x and c_y trajectories, a position control scheme for the robot joints with references obtained by inverse kinematics from these center of mass locations can be obtained. Cartesian control techniques can be applied for achieving desired CoM positions too.

However, the exact solutions the solutions for c_x and c_y consist of cosh functions (which are unbounded) and further, the result is very sensitive to the value of z_c / g [16,17]. Therefore, the application of these solutions is quite difficult. [17] also indicates similar results for fixed ZMP x-direction references and proposes an approximate solution with the use of Fourier series representation to overcome this difficulty and to obtain CoM references suitable for robust implementation of the reference generation.

Taking a similar approach, [16] develops an approximate solution for the c_x and c_y references corresponding to the moving ZMP references in Fig. 3:

$$c_x^{ref}(t) = \frac{B}{T_0}\left(t - \frac{T_0}{2}\right) + \sum_{n=1}^{\infty}\left[\frac{(B-2b)T_0^2\omega_n^2(1+\cos n\pi)}{n\pi\left(T_0^2\omega_n^2 + n^2\pi^2\right)}\sin\left(\frac{n\pi}{T_0}t\right)\right] \tag{5}$$

$$C_y^{ref}(t) = \sum_{n=1}^{\infty}\left[\frac{2AT_0^2\omega_n^2(1-\cos n\pi)}{n\pi\left(T_0^2\omega_n^2 + n^2\pi^2\right)}\sin\left(\frac{n\pi}{T_0}t\right)\right] \tag{6}$$

In this equations, ω_n stands for $\sqrt{\frac{g}{z_c}}$ The resulting c_x^{ref}, and c_y^{ref} trajectories can be seen in Figures 4 and 5, respectively.

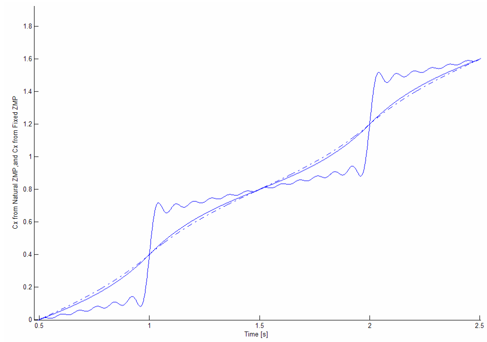


Fig. 4. c_x^{ref} (smooth curve) and p_x^{ref} (inclined staircase) trajectories for x-axis, approximate solutions.

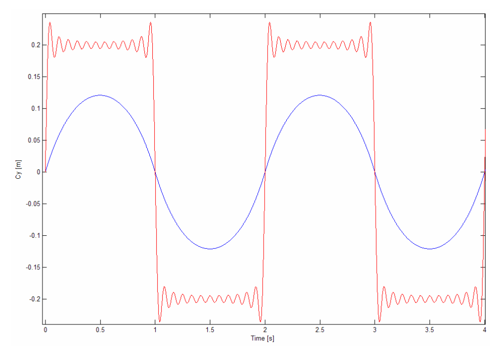


Fig.5. c_y^{ref} (sinusoidal) and p_y^{ref} (square wave) trajectories for y-axis. approximate solutions.

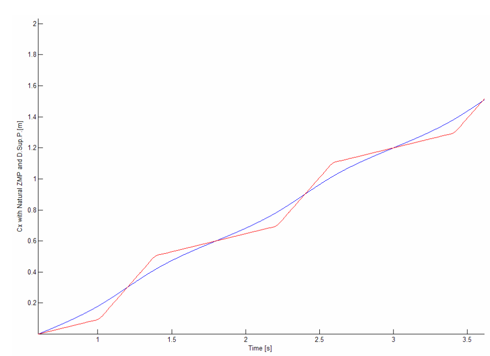


Fig. 6. c_x^{ref} (smooth curve) and p_x^{ref} (inclined staircase) trajectories for x-axis, approximate solutions after Lanczos sigma factor smoothing.

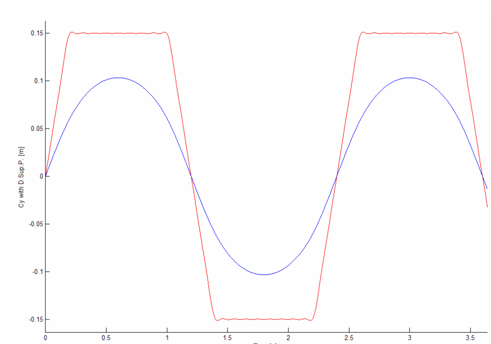


Fig. 7. c_y^{ref} (sinusoidal) and p_y^{ref} (square wave) trajectories for y-axis. approximate solutions after Lanczos sigma factor smoothing.

The ZMP reference moving forward under the sole is introduced by the discussion above. However, still the double support phase is missing. The introduction of the double support phase is achieved by a smoothing function in this paper. The Lanczos Sigma factors [21] are employed as a method to solve the Gibbs phenomenon [22], that is non-uniform convergence of the Fourier series (evident from the peaks at the discontinuities of the approximated functions). In this paper, however, the application of the Lanczos sigma factors has the second role of smoothing the abrupt changes in the reference ZMP functions and creating double support phases.

The expressions (5) and (6) with Fourier series after the Lanczos Sigma smoothing can be written as

$$C_x^{ref}(t) = \frac{B}{T_0}\left(t - \frac{T_0}{2}\right) + \sum_{n=1}^{\infty}\left[(B-2b)\frac{BT_0^2\omega_n^2(1+\cos n\pi)}{n\pi\left(T_0^2\omega_n^2 + n^2\pi^2\right)}\operatorname{sinc}\left(\frac{n\pi}{d}\right)\sin\left(\frac{n\pi}{T_0}t\right)\right] \tag{7}$$

$$C_y^{ref}(t) = \sum_{n=1}^{\infty}\left[\frac{2AT_0^2\omega_n^2(1-\cos n\pi)}{n\pi\left(T_0^2\omega_n^2 + n^2\pi^2\right)}\operatorname{sinc}\left(\frac{n\pi}{d}\right)\sin\left(\frac{n\pi}{T_0}t\right)\right] \tag{8}$$

The action of this smoothing mechanism is weighting the Fourier coefficients, and the higher the frequencies the lower are the weighting gains. The Lanczos Sigma Factor are defined by $\operatorname{sinc}(n\pi/d)$, where d is a parameter adjusting the duration of the double support phase.

Figures 6 and 7 show the effects of the Lanczos sigma factors on the x and y-direction ZMP and CoM references. This concludes the reference generation section. The next section discusses coordination and walking control.

III. COORDINATION AND CONTROL

The swing foot position references are obtained from ZMP and CoM references (Fig. 8). The control algorithm consists of five lower level position and force controller building blocks (Fig. 9). Swing foot references, or alternatively, the swing timing determines the timing for switching between control structures. However, swing reference timing is not the only criterion to switch from one control mode to the other. Switching from swing to support controller before actually reaching the ground level and establishing stable contact with the ground can cause a sudden loss of the robot balance. Therefore, ground interaction force information is used and controller mode switching is not allowed before the z-direction component of the contact force exceeds a certain threshold value. The force threshold value is a design parameter. The support-to-swing switching times are according to the swing timing without additional feedback from ground interaction forces. The double support controller regards the biped robot as a trunk manipulated by two six-DOF arms with their bases positioned on the ground level (Fig. 10, left). The CoM position reference discussed above and fixed orientation reference with respect to the world coordinate frame are applied in a position control schemes for both manipulators.

The position controllers running for the two manipulators (legs) are identical. Cartesian position and orientation errors are computed from the reference and actual position and orientations. These errors are reflected to the joint space errors by the use of inverse Jacobian relations. Independent joint PID controllers are employed for the joint space position control. The controllers for the two legs work almost independently. However, the Cartesian errors are scaled with different gains for the two legs before corresponding joint errors are computed. The scaling factor for the right leg is proportional to the horizontal distance of the left foot coordinate center from the CoM and similarly, the scaling factor for the left leg is proportional to the horizontal distance of the right foot from the CoM. This rule is obtained experimentally and it performed well for the coordination of the two legs in the double support phase. The robot in swing phases can be seen as a ground based manipulator controlling the CoM position and trunk orientation and a second manipulator based at the hip controlling the swing foot position and orientation. The right support and left swing controllers are activated simultaneously. The single support controller applies the position control scheme described above for the double support phase (without using the scaling factors). The swing leg controller is a stiffness controller for the foot position and orientation. For soft landing purposes, a Cartesian stiffness matrix with low stiffness against in orientation errors and position errors in the z-direction is employed. The horizontal directions are penalized with higher stiffness coefficients.

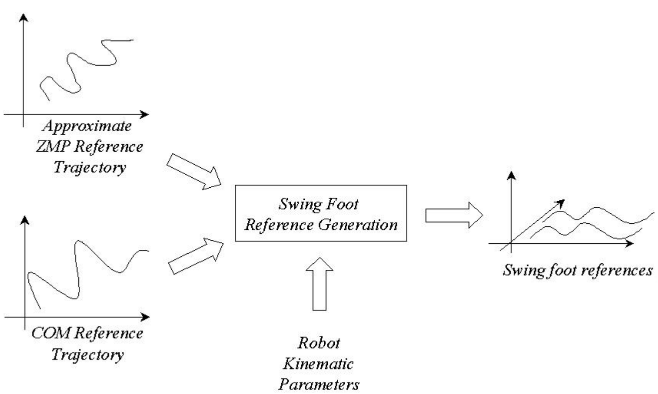


Fig. 8.The swing foot references are obtained from ZMP and CoM references.

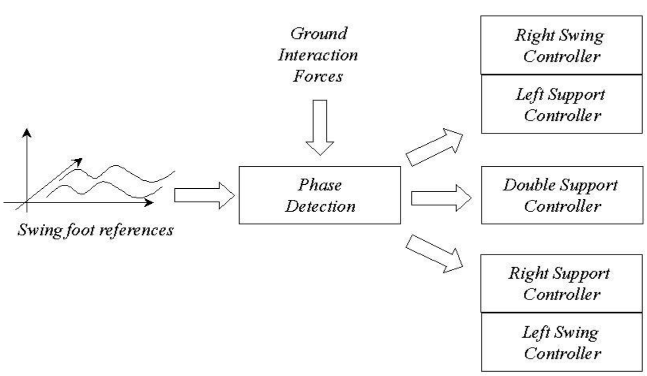


Fig. 9.The swing foot position references are obtained from ZMP and CoM references.

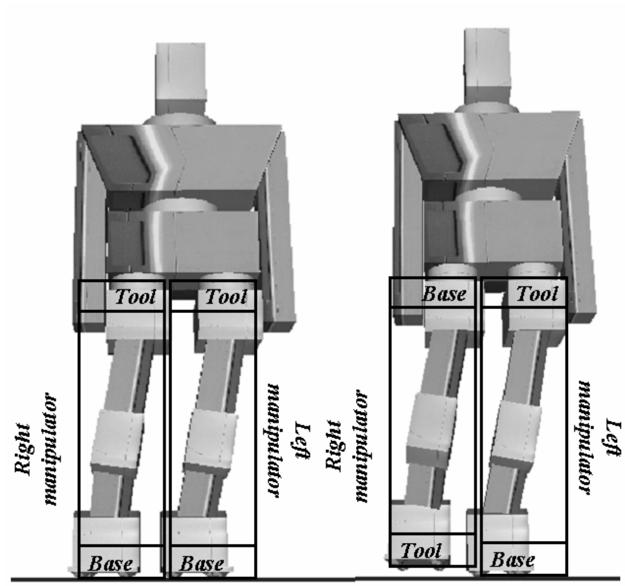


Fig. 10. The robot in the double support phase can be regarded as a trunk manipulated by two six-DOF manipulators based on the ground (left). The robot in swing phases can be seen as a ground based manipulator and a second manipulator based at the hip (right).

TABLE I
MASSES AND DIMENSIONS OF THE ROBOT LINKS

A. Link	**Dimensions (LxWxH) [m]**					**Mass [kg]**
Trunk	0.2	x	0.4	x	0.5	50
Thigh	0.27	x	0.1	x	0.1	12
Calf	0.22	x	0.05	x	0.1	0.5
Foot	0.25	x	0.12	x	0.1	5.5

TABLE II
SOME OF THE IMPORTANT SIMULATION PARAMETERS

Parameter	***Value***
Step height	0.02 m
Step period	3 s
Foot to foot y-direction distance	0.08 m
Foot to foot y-direction ZMP reference distance	0.1 m
Ground interaction threshold force	100 N

IV. SIMULATION RESULTS

The biped model used in this paper consists of two 6-DOF legs and a trunk connecting them (Fig. 1). Three joint axes are positioned at the hip. Two joints are at the ankle and one at the knee. Link sizes and the masses of the biped are given in Table I. Simulations studies are carried out with this robot model, references generated in Section II and the coordination and control mechanism discussed in Section III. The simulation scheme is similar to the one in [18]. The details of the algorithm and contact modeling can be found in [19]. Parameters used for reference generation are presented in Table II. Fig. 11 shows the y-direction CoM and CoM reference. It can be observed that the COM reference in this direction is closely tracked except in the single support phases. The y-direction ZMP and ZMP reference curves displayed in Fig. 12 also a deviation from the reference curve in the swing phases. This suggests that the simple LIMP model, concentrating on the robot trunk, and ignoring the effects of the swing foot on the CoM of the whole robot, may encounter problems when the leg weight is not very low. The legs weigh 15 kg. Although it is much less than the 50 kg trunk weight, this weight affects the y-direction COM and ZMP curves significantly. Apart from the swing phases, the tracking performance is quite acceptable. The x-direction COM and ZMP curves together with their references are presented in Figures 13 and 14, respectively. These curves, too, display oscillations and deviations from reference curves mainly due to the trunk dominated LIMP model. Still, in the average, the reference curves are tracked. In the average, the ZMP curve moves forward even in the single support phases. However, the transient behavior does not indicate that the naturalness of the human walk is achieved completely. Although there are some tracking problems as discussed above, the reference generation and control algorithms are generally successful, keeping the ZMP in the support polygon and enabling the robot move forward with an almost constant speed of 7 cm per second. This is achieved without the need for the elaborate trial and error steps common to many other reference generation approaches.

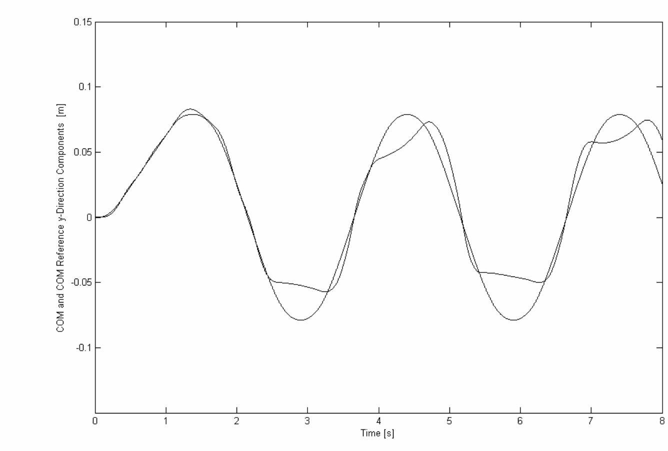


Fig. 11. CoM and CoM reference y-direction components.

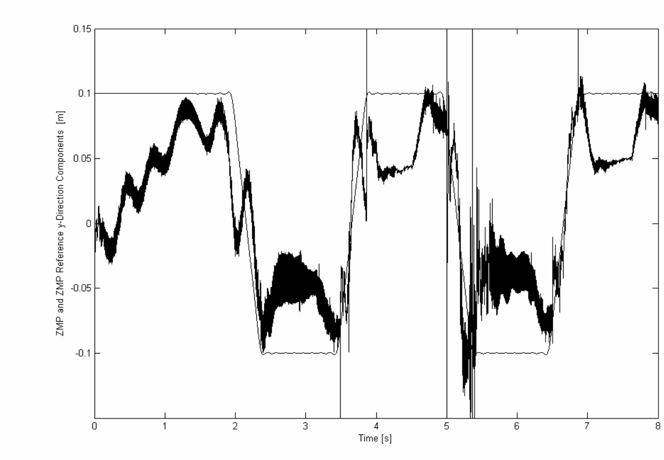


Fig. 12. ZMP and ZMP reference y-direction components

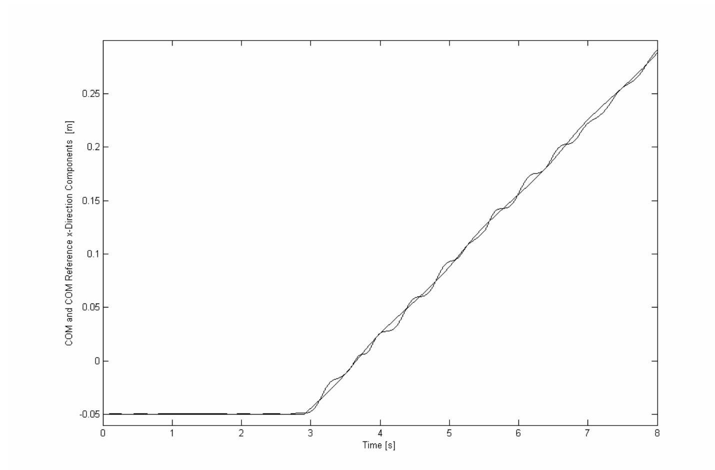


Fig. 13. CoM and CoM reference x-direction components

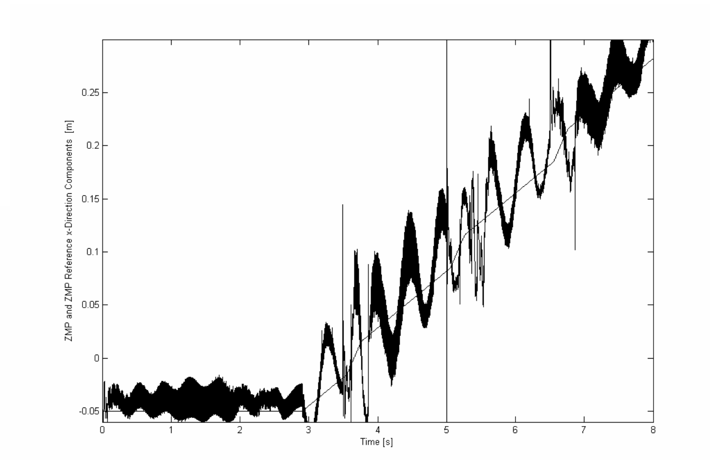


Fig. 14. ZMP and ZMP reference y-direction components

V. CONCLUSION

A trajectory generation, coordination and control approach for biped walking robots is presented. Human-like ZMP reference trajectories with Fourier series approximation techniques for the solution of LIPM dynamics equations are employed in order to achieve naturalness in the walk. A control structure consisting of different modes and position and force control techniques is employed. Simulation studies show that the reference generation without considering the effects of the heavy swing feet on robot ZMP can lead to significant deviations from reference trajectories. The walk, however, is stable and this is promising result making the algorithm a candidate for implementation.

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