Walking Pattern Generation of a Biped Walking Robot using Convolution Sum

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***Abstract*— This paper describes a novel walking pattern generation method for a biped humanoid robot using convolution sum in discrete domain. For a biped walking model, single mass inverted pendulum model is generally used and the ZMP equation is described by decoupled linear differential equation. In the viewpoint of linear system response, walking pattern generation can be regarded as a convolution of an arbitrary reference ZMP and the walking pattern for an impulse ZMP. For the calculation of convolution, the walking pattern for an impulse ZMP is derived from the analytic walking pattern for a step ZMP. Then the convolution sum is derived in two recursive forms which can be applied in online and offline respectively. The proposed algorithm requires low computation power since the walking pattern equation is composed of a recursive form. Because the algorithm is expressed in analytic form, it is not required to solve optimization problem or calculate FFT as other researchers. Computer simulation of walking also shows excellent accuracy of the proposed algorithm compared to preview control method that have been regarded as an excellent one.**

I. INTRODUCTION

HUMNOID robotics research can be classified into mobility, manipulation, navigation, intelligence, and interaction. Among these, dynamic biped walking control is related to the mobility, and it is fundamental research for realizing a humanoid robot that can move in a human environment. A realization of dynamic stability is an indispensable task for a biped walking robot. Many researchers have proposed a walking motion generation algorithm based on ZMP(Zero Moment Point)[1] to ensure the dynamic stability. The ZMP is defined as the point on the ground where the total moment of the active forces equals to zero [2]. If the ZMP is within the convex hull of all contact points between the feet and the ground, we can achieve a dynamically feasible walking motion. The practical technique on biped walking control is composed of two parts [3]. One is the walking pattern generation part which generates the feed-forward gross motion of robot and the other is online ZMP feedback control part which compensates the disturbance in real-time using sensory feedback.

In the previous development of KAIST humanoid robot, the exact solution for walking pattern was not solved at that time because the highest priority was placed at the development of online feedback controller [4-6]. In the walking pattern of KHR robot, hip trajectories are simplified into function for easy tuning in real experiment. However, walking pattern generation is as important as online ZMP feedback control. After success of walking control of KHR, the need of an accurate and flexible method on walking pattern generation arose for the walking performance. As shown in the example of a robot, Johnnie, the walking speed was improved with the help of the optimized trajectory [7].

In order to generate walking pattern, reference ZMP patterns and foot trajectories are designed first. Here, walking pattern generation means calculating the whole body motion trajectories that realize the given reference ZMP trajectory. This can be solved by lots of iterative method [8][9]. In order to guarantee the uniqueness of the solution, some kinematic constraints are imposed [8].

The research on the stable walking pattern generation is initiated in the Waseda University. Takanishi *et al.* designed the reference ZMP with the trajectories of lower and upper body and solved the 3 D.O.F trunk motion in the ZMP equation[9]. They imposed the kinematic constraints on the upper body motion to solve the interferential and nonlinear differential equation. They regarded the left and right side of the equation as the periodic function and solved the approximated periodic solution by comparing the coefficients of the right and left side of the equation using FFT. By iteration, more accurate solution could be achieved.

In the walking pattern generation method of H5 robot in Tokyo University, Nagasaka *et al.* solved the hip trajectory which realize the given reference ZMP and foot trajectory with the assumption that the hip height of a robot is constant[8]. They solved the ZMP equation iteratively using optimal gradient method but it took about 24 hours to get the convergent solution.

Kajita *et al.* introduced the single mass inverted pendulum model(cart-table model) instead of using the complicated model where multi-body mass is considered.[10] Then, they transformed it into the state equation. It is remarkable that they regarded the walking pattern generation problem as the tracking control problem which tracks the desired ZMP. To solve the tracking control problem, they proposed to use optimal preview control and finally solved the position of the center of mass, which was the walking pattern that followed a given reference ZMP trajectory. The above method has an advantage that it require less computation power compared with the methods of Waseda and Tokyo University.

Compared with the above algorithms, a novel walking pattern algorithm using convolution sum is described in this paper. It is organized as follows: In section II, the modeling for a biped robot is considered and an acausal analytic walking pattern for a step reference ZMP is solved. In section III, the walking pattern for an impulse reference ZMP is derived using the result of section II. Then, the convolution sum can be calculated from the impulse walking pattern and an arbitrary reference ZMP. Online and offline algorithm in recursive form is derived for reducing computation power. Section IV presents the simulation results of the proposed algorithm. Finally, in section V, we conclude the paper with discussion.

II. MODELLING OF A BIPED ROBOT AND ACAUSAL SOLUTION FOR A STEP REFERENCE ZMP

A. Modeling of a Biped Robot and ZMP equation

For the walking pattern generation of a biped humanoid robot, the modeling of biped walking related with ZMP is required first. From the equation of motion for an *n*-D.O.F biped robot, the ZMP equation can be obtained [11].

$$x_{zmp} = \frac{\sum m_i(\ddot{z}_i + g)x_i - \sum m_i z_i \ddot{x}_i - \sum I_{iy}\ddot{\theta}_{iy}}{\sum m_i(\ddot{z}_i + g)} \tag{1}$$

$$y_{zmp} = \frac{\sum m_i(\ddot{z}_i + g)y_i - \sum m_i z_i \ddot{y}_i - \sum I_{ix}\ddot{\theta}_{ix}}{\sum m_i(\ddot{z}_i + g)} \tag{2}$$

In the above ZMP equation, the x-component of the ZMP is independent of y-axis motion except for the inertia tensor term, and is a linear combination of x and $\ddot{x}$. This inertia tensor term can be neglected under assumption that the influence of the inertia tensor is relatively small when a robot model is made with links and the torso does not rotate [11]. In the walking pattern generation of Wabian[9] and H5[8][11], the above ZMP equation was used, which was obtained from multi-body model. It will be a good modeling for ZMP equation if we have precise information of robot dynamic parameters such as mass, location of center of gravity, moment of inertia of each link. However it is actually hard to know exactly. Equation (1) and (2) becomes (3) and (4) if we assume the following condition:

1) Robot is modeled as a single mass inverted pendulum,
2) The height of the mass center is l
3) Inertia tensor term is neglected.

$$x_{zmp} = \frac{mgx - ml\ddot{x}}{mg} = x - \frac{l}{g}\ddot{x} = x - \frac{1}{\lambda^2}\ddot{x} \tag{3}$$

$$y_{zmp} = y - \frac{1}{\lambda^2}\ddot{y} \tag{4}$$

$$\text{where } \lambda = \sqrt{\frac{g}{l}} \tag{5}$$

Kajita *et al.* used the above single mass inverted pendulum model [10]. In order to make the ZMP equation simply, it is general to preserve the height of pelvis. The simple modeling of Kajita brought a successful result in the real experiment and the maximum walking speed of HRP-2 was 2.5 km/h [12]. Although algorithm is developed under the assumption of a single mass inverted pendulum model, it can be applied on the multi body model as in [10]. So the model of a biped robot is assumed as a single mass inverted pendulum in this paper.

B. Walking Pattern Generation

By substituting the trajectory of the center of mass into (3) and (4), the resulted ZMP can be calculated easily and directly. On the other hand, walking pattern generation is to find the CoM(Center of Mass) which result in the given reference ZMP, and it corresponds to inverse problem of the direct calculation [10]. Its solution is not solved easily. For the generality, walking pattern generation algorithm should be applied to the arbitrary reference ZMP. The requirement of good walking pattern generation algorithm can be summarized as follows:

1) Accuracy and simplicity applicable to any kinds of biped robot easily.
2) Low computation power and short calculation time
3) Algorithm should not diverse, and it should guarantee the convergence.
4) Flexibility : Parameters such as walking period and step length can be changed freely online.

In this paper, it is shown that the proposed method satisfies at least the first three requirements. In the novel algorithm proposed in this paper, analytic walking pattern is first considered for the step reference ZMP instead of arbitrary reference ZMP. That is to solve the trajectory of center of mass x in order to change ZMP with step function from 0 to 1 at some specific instant. The analytical solution is solved for this problem in section II.C, and the solution is extended to be applicable to arbitrary reference ZMP in section III.

C. Acausal Analytic Solution for a Step ZMP input

When a biped robot walks stably, it shifts its ZMP periodically from one foot to the other foot and its center of mass also moves accordance with ZMP shift. In this viewpoint, the meaningful and basic shape of reference ZMP will be a step function. So, let's set the reference ZMP into unit step function that changes value from 0 to 1 at t_0 as shown in (6) and find the solution x that satisfy the equation (3) by analytical method.

$$x_{zmpref} = 1(t - t_0) = \begin{cases} 0, & t < t_0, \\ 1, & t \geq t_0 \end{cases} \tag{6}$$

The homogeneous solution of ZMP equation is solved from the corresponding characteristic equation (7)

$$1 - \frac{1}{\lambda^2}s^2 = 0 \tag{7}$$

Its solution yields the characteristic roots:

$$s_{1,2} = \pm\lambda \tag{8}$$

So, the general solution is

$$y = C_1 e^{\lambda t} + C_2 e^{-\lambda t} \tag{9}$$

Among the causal solution for the step ZMP input, there is no solution that does not diverse and is continuous. While causal solution depends only on past and present values of input, acausal solution depends on the past, current and future input values. If stable and continuous solution is considered among the acausal solution with the idea of equation (9), the stable solution that satisfies (3) and (6) is represented as:

$$x(t) = \begin{cases} \frac{1}{2} e^{\lambda(t-t_0)} & ,t < t_0 \\ 1 - \frac{1}{2} e^{-\lambda(t-t_0)} & ,t \geq t_0 \end{cases} \tag{10}$$

where x(t) is monotonically increasing function, and differentiable and continuous at t_0, so it satisfy the following equations.

$$\lim_{t \to t_0+0} x(t) = \lim_{t \to t_0-0} x(t) = 0.5 \tag{11}$$

$$\lim_{t \to t_0+0} \dot{x}(t) = \lim_{t \to t_0-0} \dot{x}(t) = \frac{\lambda}{2} \tag{12}$$

The graph of analytic solution (10) for a step reference ZMP is shown in figure 1, where g is 9.81m/sec^2, and l is 0.8 m.

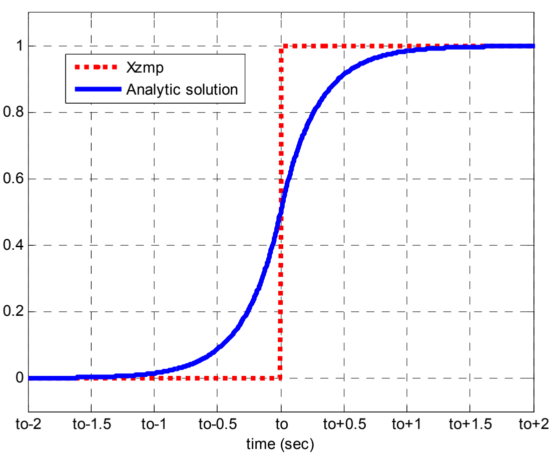

Fig. 1. Acausal analytic solution (CoM profile) for a step reference ZMP

The validity of the solution (10) is confirmed by substituting it into the ZMP equation (3). By substituting (10) in (3), we can verify that this solution satisfies the given ZMP equation. The resulted ZMP is calculated as follows, and that is a step function described in (6).

for $t < t_0$,

$$x_{zmp}(t) = 0.5 e^{\lambda(t-t_0)} - \frac{1}{\lambda^2}\left(0.5\lambda^2 e^{\lambda(t-t_0)}\right) = 0 \tag{13}$$

for $t \geq t_0$,

$$x_{zmp}(t) = \left(1 - 0.5 e^{-\lambda(t-t_0)}\right) - \frac{1}{\lambda^2}\left(-0.5\lambda^2 e^{-\lambda(t-t_0)}\right) = 1$$

III. WALKING PATTERN GENERATION USING CONVOLUTION SUM

A. *Impulse response for an impulse ZMP input*

Output of a linear system for an arbitrary input can be calculated from its impulse response by the convolution integral.

$$y(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau) d\tau \tag{14}$$

Here, $h(t\text{-}\tau)$ means impulse response at t to an impulse applied at τ. That is, the response $y(t)$ is given by a weighted sum over the entire time history of the input $u(t)$. In the discrete domain, the following convolution sum is valid.

$$y[n] = u[n] * h[n] = \sum_{k=-\infty}^{k=\infty} u[k] h[n-k] \tag{15}$$

Once the solution $h(t)$ of ZMP equation for an impulse reference ZMP is known, the trajectory $x(t)$ that realize an arbitrary reference ZMP $u(t)$ can be calculated by (15). It is hard to solve the impulse response of ZMP equation directly, but the impulse response can be easily derived from the step response described in the previous section.

If $u_s[k]$ is a step function, where $u_s[k]$ =1 for $k \geq 0$ else $u_s[k]$ =0, the step response s[n] is represented as:

$$s[n] = u_s[n] * h[n] = \sum_{k=-\infty}^{k=\infty} u_s[k] h[n-k] = \sum_{k=-\infty}^{n} h[k] \tag{16}$$

Here, subtracting $s[n\text{-}1]$ from $s[n]$ yields impulse response $h[n]$.

$$s[n] - s[n-1] = \sum_{k=-\infty}^{n} h[k] - \sum_{k=-\infty}^{n-1} h[k] = h[n] \tag{17}$$

When the step is applied at t_0=0, discrete form of the step response equation (10) is described by

$$s[n] = \begin{cases} \frac{1}{2} e^{\lambda n T} & , \; n < 0 \\ 1 - \frac{1}{2} e^{-\lambda n T} & , \; n \geq 0 \end{cases} \tag{18}$$

where T is a sample period. Substituting (18) into (17) leads to the discrete form of the impulse response $h_{nonsym}[n]$.

$$h_{nonsym}[n] = \begin{cases} \frac{1}{2}(e^{\lambda T} - 1) e^{\lambda(n-1)T} & , \; n < 1 \\ \frac{1}{2}(e^{\lambda T} - 1) e^{-\lambda n T} & , \; n \geq 1 \end{cases} \tag{19}$$

In (19), the peak of $h[n]$ exists on both $h[1]$and $h[0]$ and it is not symmetric about the zero point. If we consider the impulse response function where the midpoint of an impulse is exactly applied at 0, the h[n] is expressed as follows.

$$h[n] = h[0] \cdot e^{-|n|\lambda T}$$

$$where \; h[0] = \frac{1}{2}(e^{\lambda T} - 1) e^{-\frac{\lambda T}{2}} = \sinh\left(\frac{\lambda T}{2}\right) \tag{20}$$

This impulse response function has the following properties, and they are represented as:

$$h[n] = h[-n] \tag{21}$$

$$h[n+1] = e^{-\lambda T} h[n] \qquad , \; n \ge 0$$
$$where \;\; 0 < e^{-\lambda T} < 1 \tag{22}$$

Impulse response, $h[n]$ is plotted in figure 2, where g is $9.81 m/sec^2$, l is 0.8 m, and the sample period T is 0.005 sec. Here, $h[n]$ is non-causal since $h[n] \neq 0$ for all $n < 0$. Non-causal property of impulse response is inherited from the step response. It is non-causal but stable since the following equation is assured.

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \tag{23}$$

Regardless of the non-causal property of impulse response function, the walking pattern $x(t)$ that realize an arbitrary reference ZMP can be calculated by convolution sum.

In reference [13], Lim and Takanishi performed the trunk motion simulation with their iterative pattern generation method. At the simulation, an impulse moment is applied to the pitch trunk when a biped robot is not in motion. In the result of iterative method, compensatory trunk motion was generated before and after the impulse moment is applied to the biped robot. The graph of converged the pitch motion was strikingly similar to analytical $h[n]$ of figure 2. So, this fact also confirms the validity of the impulse response function $h[n]$.

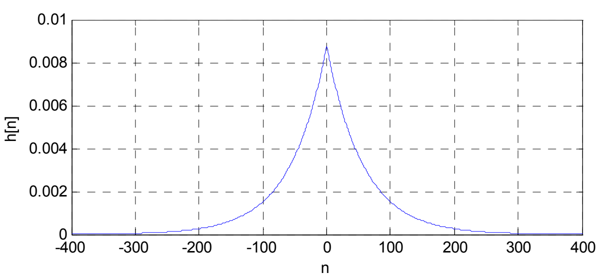

Fig. 2. Acausal impulse response function for an impulse reference ZMP

B. Derivation of online walking pattern generation algorithm using convolution sum

Using $h[n]$ in (20), walking pattern $x[n]$ that realize the arbitrary reference ZMP $u[n]$ can be obtained by substituting $u[n]$ into (15). Even though we take the summation until sufficiently large number N in (15), the resulted ZMP is approximately equal to original convolution sum as follows:

$$x[n] = \sum_{k=-\infty}^{k=\infty} u[n-k]h[k] \approx \sum_{k=-N}^{k=N} u[n-k]h[k] \tag{24}$$

Selection of N influences on the accuracy of ZMP and computation power. If N is large, the resulted ZMP is accurate but it requires more computation power.

In order to reduce the calculation time, some part of (24) can be calculated in a recursive way. Using the property of (21), we get the following $x[n]$ which is a sum of future term and past term.

$$\begin{aligned} x[n] &= \sum_{k=-\infty}^{k=\infty} u[n-k]h[k] \\ &= \sum_{k=-\infty}^{k=0} u[n-k]h[k] + \sum_{k=1}^{k=\infty} u[n-k]h[k] \\ &= \sum_{k=1}^{k=\infty} u[n+k-1]h[k-1] + + \sum_{k=1}^{k=\infty} u[n-k]h[k] \end{aligned} \tag{25}$$

$$x[n] = x_{future}[n] + x_{past}[n] \tag{26}$$

$$\text{where} \quad x_{future}[n] = \sum_{k=1}^{k=\infty} u[n+k-1]h[k-1]$$
$$x_{past}[n] = \sum_{k=1}^{k=\infty} u[n-k]h[k]$$

Using (20), we get the past term:

$$\begin{aligned} x_{past}[n] &= \sum_{k=1}^{k=\infty} u[n-k] \cdot h[0] \cdot e^{-|k|\lambda T} \\ &= \{u[n-1] \cdot e^{-\lambda T} + u[n-2] \cdot e^{-2\lambda T} \\ &\quad + u[n-3] \cdot e^{-3\lambda T} + \cdots\} \cdot h[0] \end{aligned} \tag{27}$$

$$\begin{aligned} x_{past}[n-1] &= \{u[n-2] \cdot e^{-\lambda T} + u[n-3] \cdot e^{-2\lambda T} \\ &\quad + u[n-4] \cdot e^{-3\lambda T} + \cdots\} \cdot h[0] \end{aligned} \tag{28}$$

From (27) and (28), the recursive form of the past term is obtained.

$$x_{past}[n] = e^{-\lambda T} x_{past}[n-1] + h[0] \cdot e^{-\lambda T} u[n-1] \tag{29}$$

Online recursive form for the future term where $x_{future}[n]$ is calculated using $x_{future}[n\text{-}1]$ may be derived, but it cannot be applicable because of the possibility of divergence. If we summarize the online walking pattern generation algorithm using convolution sum, then

$$\begin{aligned} x[n] &= x_{future}[n] + x_{past}[n] \\ &= \sum_{k=1}^{k=N} u[n+k-1]h[k-1] + x_{past}[n] \end{aligned} \tag{30}$$
$$x_{past}[n] = e^{-\lambda T} x_{past}[n-1] + h[0] \cdot u[n-1]$$
$$h[n] = h[0] \cdot e^{-|n|\lambda T}$$
$$where \;\; h[0] = \sinh\left(\frac{\lambda T}{2}\right) \; , \;\; \lambda = \sqrt{g/l}$$

The above proposed algorithm can be applied in online calculation, but it use a finite number of future ZMP reference. That is similar to the preview control of Kajita [10]. $x_{future}[n]$ plays the similar role of the preview term, and $x_{past}[n]$ resembles the tracking control term. In the preview control of Kajita, there is a complicated difficulty that the Riccati equation has to be solved to decide the preview gain G_P and the feedback gain G_X after selecting Q and R. As the height of mass center L is changed, the above procedure should be repeated again. However, in the approach of convolution sum, analytical impulse response function is given in terms of L. The corresponding terms of G_P and G_X are analytically calculated without solving the Riccati equation. This is the advantage of proposed algorithm over the walking pattern generation algorithm using preview control.

C. Derivation of offline walking pattern generation algorithm using convolution sum

With the same procedure of the past term, the future term can be derived in offline recursive form that can be used in batch process.

$$x_{future}[n] = e^{-\lambda T} x_{future}[n+1] + h[0]u[n] \qquad (31)$$

In this equation, $x_{future}[n]$ is calculated using $x_{future}[n+1]$. Since it uses future value to calculate the current value, it cannot be applied in online, but in offline. If the recursive form (31) is used instead of convolution form in batch process, the computation power is remarkably reduced.

If the number of data for walking pattern is M, and the center of mass does not accelerated anymore at Mth end point, then the position of CoM will be identical to the reference ZMP at M. From (3) and (26), we get

$$x[M] = x_{future}[M] + x_{past}[M] = u[M] \qquad (32)$$

So, the initial value of the recursive form for the future term, $x_{future}[M]$ is obtained by

$$x_{future}[M] = u[M] - x_{past}[M] \qquad (33)$$

Ideally, $x_{future}[M]$ and $x_{past}[M]$ are a half of u[M] when the center of mass does not move and the reference ZMP is not changing at the end of walking. This fact will be shown in the simulation result of next section. Finally, the walking pattern generation in the batch process is summarized as

$$\begin{aligned} &x[n] = x_{future}[n] + x_{past}[n] \\ &where \quad x_{past}[n] = e^{-\lambda T} x_{past}[n-1] + h[1]\cdot u[n-1] \quad (34) \\ &\qquad\qquad x_{future}[n] = e^{-\lambda T} x_{future}[n+1] + h[1]\cdot u[n] \end{aligned}$$

The proposed offline walking pattern generation algorithm using convolution sum has the following advantages.

1) *Low computation*: To get the walking pattern with equation (34) in the batch process, it is only required to calculate the simple recursive equation 2M times for the given M number of reference ZMP data. In addition, there is no burden to compute the FFT as the algorithm of Lim and Takanish [13], where it assumes the periodic solution.

2) *Good accuracy*: Since the offline equation (34) is derived from the convolution sum counting the infinite number, it is more accurate compared with the preview control in [10] where the finite number of preview length is used.

3) *Applicable to the multi-body model*: Although the proposed algorithm considered a single mass inverted pendulum model, it also can be applied to the multi-body dynamics in the batch process by applying the algorithm iteratively like a preview control in [10]. Proposed algorithm still requires low computation compared to the walking pattern generation method using FFT, inverse FFT, and iterative rule for the muli-body model [13].

IV. WALKING SIMULATION FOR AN ARBITRARY ZMP

In order to investigate the characteristics of the proposed walking pattern generation algorithms and evaluate their performances, walking simulation was performed. The height of the center of mass was set to 0.814m. The trajectory of ZMP reference is given with the thin dotted line as shown in Fig. 3, where the robot walks 3 steps. The coordinate X and Y denote the sagittal and frontal direction respectively. In order to compare the performance and verify the validity, same reference ZMP was adopted from [10] where preview control method was simulated. In the figure, the step length is 300 mm, the step period is 0.8 sec, and duty factor is 0.55.

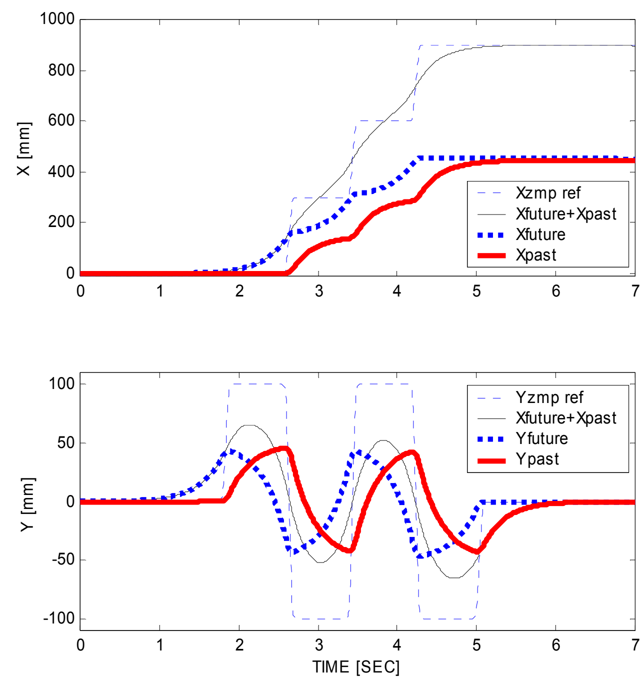

Fig. 3 Trajectory of a CoM obtained by online walking pattern algorithm using convolution sum.

The calculated ZMP obtained by online convolution algorithm of equation (30) is the sum of the future and past term as shown in Fig. 3. The future term starts to change before the change of the reference ZMP and the past term begins to respond after the change of reference ZMP. In this online process, the future term uses the future 1.6 second reference ZMP where N is 320 and sampling period is 0.005sec. We can see that both x_{future} and x_{past} is a half of the reference ZMP at the end of walking. In addition, the profile of the future term on the Y axis is similar to that of the past term when time goes backward. To compare the accuracy of the ZMP tracking, the online and offline algorithms are applied to the same reference ZMP with preview control method. Fig. 4 shows the good tracking performance for all three methods. In the walking pattern generation method using preview control, the preview length was set to 1.6 second and preview gain G_p and state feedback gain G_x were calculated from the Riccati equation by setting R=1.0×10^{-6}, Q_e=1.0, Q_x=0. [10] The integral gain G_i was not considered in this simulation because of the possibility of divergence.

The ZMP error of the offline algorithm using convolution sum is smallest among them. The average deviation of the calculated ZMP from the reference is about 0.018 mm. It is accurate because the simple recursive form of the offline equation (34) includes the infinite number of summation. It can be regarded as a perfect walking pattern generator.

The average ZMP error of the online algorithm using convolution sum is 1.351mm, which is the smaller than 2.619mm of preview control. In Fig. 4, the main ZMP error of the online algorithm using convolution sum resulted from the future term since its peak of the ZMP error arises before 1.6 second transition of the reference ZMP.

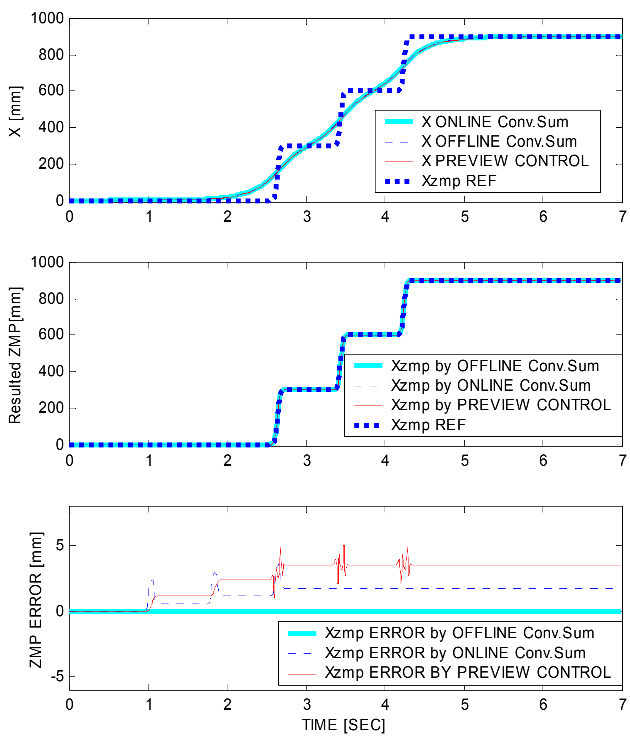

Fig. 4. Comparison of ZMP error for three algorithms

V. Conclusion

In this paper, novel online and offline walking pattern generation algorithm using convolution sum were proposed. Biped walking robot was modeled as a single inverted pendulum model, and the linear decoupled differential ZMP equation was solved by convolution sum. Noncausal impulse response was derived from the step response function and the convolution equation was derived in recursive form for fast computation. It also brought the accuracy of the solution since infinite sum of the convolution could be involved in the recursive form. Proposed online algorithm has a similarity with the preview control which needs a finite future reference, but it is not required to solve Riccati equation to get gains. It has an advantage that it is only required to calculate the analytical impulse response function to calculate the walking pattern. The great accuracy of the proposed offline walking pattern algorithm was confirmed in the walking simulation by comparing the result of other methods. The proposed algorithm can be also applied to the multi-body model by applying it iteratively. In the application of biped walking, it is expected that the proposed algorithms will be widely used because equations of algorithms are simple and accurate walking pattern can be achieved.

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