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On the Walking Control for Humanoid Robot based on the Kinematic Resolution of CoM Jacobian with Embedded Motion

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***Abstract*— This paper proposes the walking pattern generation method, the kinematic resolution method of CoM(center of mass) Jacobian with an embedded motion, and the walking controller design method for humanoid robots. First, the walking pattern is generated using the simplified model for bipedal robot. Second, the kinematic resolution of CoM Jacobian with embedded motion makes a humanoid robot balanced automatically during the movement of the all other limbs. Actually, it offers the ability of whole body coordination to the humanoid robot. Third, the walking controller is composed of the CoM controller minus the ZMP(zero momentum position) controller. Also, we show that the proposed walking controller brings the disturbance input-to-state stability (ISS) for the simplified bipedal walking robot model. Finally, the effectiveness of the proposed kinematic resolution method and walking controller is shown through experiments in regard to humanoid robot dancing and walking.**

I. Introduction

Recently, there have been many researches about humanoid motion control, for example, walking control [1]–[6], running control [7] and whole body coordination [8], [9]. Especially, the whole body coordination (WBC) algorithm with good performance becomes the essential part in the development of humanoid robot because it offers the enhanced stability and flexibility to the humanoid motion planning. In this paper, we suggest the kinematic resolution method of CoM Jacobian with embedded (walking or dancing) motion, actually, which offers the ability of WBC to the humanoid robot. For example, if the humanoid robot stretches two arms forward, then the position of CoM(center of mass) of humanoid robot moves forward and its ZMP(zero momentum point) swings back and forth. In this case, the proposed kinematic resolution method of CoM Jacobian with embedded motion offers the configurations of supporting limb(s) which is(are) calculated automatically to maintain the position of CoM fixed at one point.

Also, the walking controller design with good performance becomes the important part in the development of humanoid robot. In the walking control, the ZMP control is the most important factor in implementing stable bipedal robot walking or dancing. If the ZMP is located in the region of supporting sole, then the robot will not fall down during walking. To implement stable robot walking, the desired ZMP planning methods were first suggested by using the inverted pendulum model [2] and the fast fourier transformation [5]. In order to compensate the error between the desired and actual ZMP, various ZMP control methods were suggested: for example, direct/indirect ZMP control methods [1], [3], [10] and the impedance control [4]. Despite many references to bipedal walking control methods, research on the stability of bipedal walking controllers is still lacking. The exponential stability of periodic walking motion was partially proved for a planar bipedal robot in [6]. Also, the ISS of the indirect ZMP controller was proved for the simplified bipedal robot model in [1]. In this paper, we will propose the walking control method and prove its ISS. Due to the modelling uncertainties and the complexity of the full dynamics for a bipedal walking robot, we will represent the dynamic walking robot as a simple rolling sphere model on a constraint surface.

This paper is organized as follows: section II introduces a simplified model for a bipedal walking robot, section III proposes the walking pattern generation method by planning the desired CoM trajectory from the desired ZMP trajectory, section IV explains the kinematic resolution method of CoM Jacobian with an embedded walking or dancing motion, section V proves the ISS of the proposed walking control for the simplified bipedal robot model, section VI shows the experimental results about the stable robot walking and the WBC functions obtained by using the kinematic resolution method of CoM Jacobian with embedded walking and dancing motions, and section VII concludes the paper.

II. Simplified Model for Bipedal Robot

The bipedal walking mechanism is an essential part of humanoid as shown in Fig. 1. Since humanoid legs have high degrees of freedom for human-like walking, it is difficult to use their dynamics to design controller and to analyze stability. Therefore, we will simplify the walking related dynamics of bipedal robot as the equation of motion for a point mass at CoM.

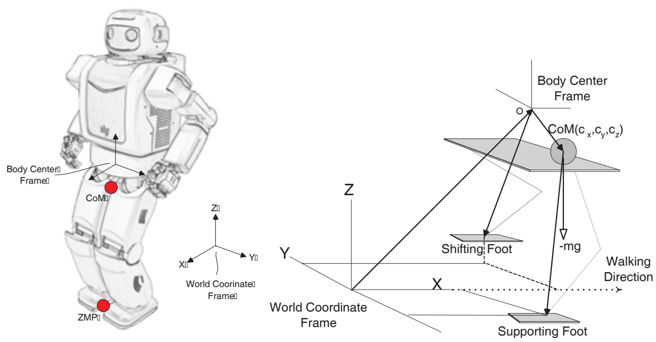

Fig. 1. Rolling Sphere Model for Dynamic Walking

First, if we assume that the motion of CoM is constrained on the surface $z = c_z$, then the rolling sphere model with the concentrated point mass m can be obtained as the simplified model for bipedal robot as shown in Fig. 1. In this figure, the motion of the rolling sphere on a massless plate is described by the position of CoM, $\boldsymbol{c} = [c_x, c_y, c_z]^T$, and the ZMP is described by the position on the ground, $\boldsymbol{p} = [p_x, p_y, 0]^T$. In the robot walking motion, the joint configurations of supporting leg are firstly determined by using the kinematic resolution method of CoM Jacobian with the embedded walking motion and those of the shifting leg are secondly determined by solving the inverse kinematics for the shifting leg motion expressed in world coordinate frame. These will be explained in the following section.

Second, from the equations of motion of the rolling sphere (mass = m) expressed on the plane $z = c_z$ in Fig. 1, the ZMP equations can be obtained as two differential equations:

$$p_x = c_x - \frac{c_z}{g}\ddot{c}_x \tag{1}$$

$$p_y = c_y - \frac{c_z}{g}\ddot{c}_y. \tag{2}$$

The state space realization of ZMP equations (1) and (2) can be written as:

$$\frac{d}{dt}\begin{bmatrix} c_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \omega_n^2 & 0 \end{bmatrix}\begin{bmatrix} c_i \\ \dot{c}_i \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega_n^2 \end{bmatrix} p_i, \tag{3}$$

for $i = x, y$, where $\omega_n = \sqrt{g/c_z}$ and g is the gravitational acceleration constant. These state space equations describe the relation between the position of CoM and the ZMP, and they will be used to prove the stability of the walking controller in the following section.

III. Desired ZMP/CoM Trajectories

To implement robot walking, first of all, the stepping positions on the ground and the supporting phases are predetermined as shown in Fig. 2. In this figure, the stepping positions are generally represented as periodic functions and the supporting phases (double supporting and single supporting) are used in moving the ZMP. In a single supporting phase, the ZMP should stay in the sole of supporting leg while the shifting leg is making a step. In a double supporting phase, the ZMP should be moved to the sole of shifting leg. These procedures should be repeated to make stable robot walking. Also, the desired trajectory of CoM should be derived from the desired ZMP in Fig. 2. In this section, we develop the equations for the generation of the desired trajectories of CoM.

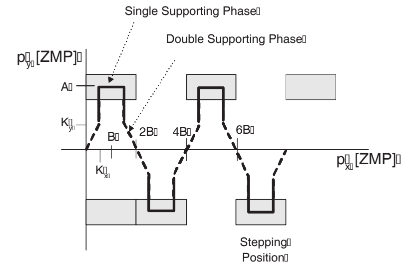

Fig. 2. Desired ZMP Trajectory

A. X Directional Planning

The X-directional ZMP trajectory in Fig. 2 is expressed with a period time T as following forms:

$$\begin{aligned} &\text{for} \quad 0 \le t < t_d, && p_x(t) = (K_x/t_d)t = c_x(t) \\ &\text{for} \quad t_d \le t < T - t_d, && p_x(t) = B \\ &\text{for} \quad T - t_d \le t < T, && p_x(t) = (2B - K_x) \\ & && +(K_x/t_d)(t - (T - t_d)) = c_x(t), \end{aligned} \tag{4}$$

where B is the half of step length and t_d means the time when $p_x(t_d) = K_x$ in the ZMP graph of broken line, namely, the change time t_d from the double supporting phase to single supporting one. Here, the desired trajectory of CoM should be determined from Eq. (1) with Eq. (4), in other words, we should solve the following differential equation for $t_d \le t < T - t_d$:

$$\ddot{c}_x - \omega_n^2 c_x = -\omega_n^2 B.$$

The general solution is obtained as:

$$c_x(t) = C_{x1}\cosh(\omega_n(t - t_d)) + C_{x2}\sinh(\omega_n(t - t_d)) + B, \tag{5}$$

with the unknown coefficients C_{x1} and C_{x2}. The unknown coefficients satisfying the following boundary conditions

$$\begin{aligned} c_x(t_d) &= K_x \\ \dot{c}_x(t_d) &= K_x/t_d, \end{aligned}$$

can be determined as :

$$C_{x1} = K_x - B \tag{6}$$

$$C_{x2} = \frac{K_x}{t_d\omega_n}. \tag{7}$$

Also, for $c_x(T - t_d) = 2B - c_x(t_d)$ and $\dot{c}_x(t_d) = \dot{c}_x(T - t_d)$, the following constraint equation should be always satisfied:

$$K_x = \frac{Bt_d\omega_n}{t_d\omega_n + \tanh\left(\omega_n\left(\frac{T}{2} - t_d\right)\right)}. \tag{8}$$

Therefore, if we arbitrarily determine the change time t_d of supporting phases with the positive constants ω_n and B, then

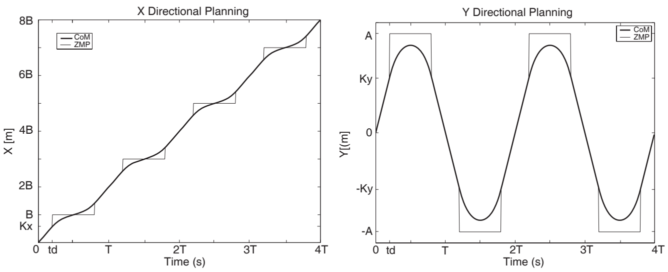

Fig. 3. X- and Y-Directional Desired ZMP/CoM Trajectories

the K_x is determined from Eq. (8), and then, the unknown coefficients C_{x1} and C_{x2} can be determined from Eq. (6) and (7). Now, the X-directional desired trajectory of CoM can be obtained by the smooth function as schematically depicted in Fig. 3.

B. Y Directional Planning

The Y-directional ZMP trajectory in Fig. 2 is also described as following forms:

$$\begin{array}{lll} \text{for} \quad 0 \le t < t_d, & p_y(t) = (K_y/t_d)t = c_y(t) \\ \text{for} \quad t_d \le t < T - t_d, & p_y(t) = A \\ \text{for} \quad T - t_d \le t < T, & p_y(t) = -(K_y/t_d)(t-T) = c_y(t) \end{array} \tag{9}$$

where A is the half of the distance between both feet and t_d means the time when $p_y(t_d) = K_y$ in the ZMP graph of broken line. Here, the desired trajectory of CoM should be obtained by solving the following differential equation :

$$\ddot{c}_y - \omega_n^2 c_y = -\omega_n^2 A, \quad \text{for} \;\; t_d \le t < T - t_d$$

The general solution is also obtained as:

$$c_y(t) = C_{y1}\cosh(\omega_n(t-t_d)) + C_{y2}\sinh(\omega_n(t-t_d)) + A \tag{10}$$

with the unknown coefficients C_{y1} and C_{y2}. The unknown coefficients satisfying the following boundary conditions

$$\begin{aligned} c_y(t_d) &= K_y \\ \dot{c}_y(t_d) &= K_y/t_d, \end{aligned}$$

can be also determined as:

$$C_{y1} = K_y - A \tag{11}$$

$$C_{y2} = \frac{K_y}{t_d\omega_n}. \tag{12}$$

Also, for $c_y(t_d) = c_y(T - t_d)$ and $\dot{c}_y(t_d) = -\dot{c}_y(T - t_d)$, the following constraint equation should be always satisfied:

$$K_y = \frac{A t_d \omega_n \tanh\left(\omega_n\left(\frac{T}{2} - t_d\right)\right)}{1 + t_d\omega_n \tanh\left(\omega_n\left(\frac{T}{2} - t_d\right)\right)}. \tag{13}$$

Therefore, from the determined t_d, ω_n and A, the K_y is determined from Eq. (13), and then, the unknown coefficients C_{y1} and C_{y2} are determined from Eq. (11) and (12). The Y-directional desired trajectory of CoM can be obtained by the smooth function as schematically depicted in Fig. 3.

To implement the desired CoM motions of Fig. 3 in the real humanoid robot, the CoM inverse kinematics is required to resolve them kinematically according to the driving motor axes. That is derived from the CoM Jacobian between the velocity of CoM and the joint velocity of the supporting leg(s). The concrete resolution method will be explained in the following section.

IV. Kinematic Resolution of CoM Jacobian with an Embedded Motion

Let us derive the partitioned CoM Jacobian to embed a desired motion. Let a robot has n limbs and the first limb be the base limb. The base limb can be any limb but it should be on the ground to support the body. Each limb of a robot is considered as an independent limb, hereafter. In general, the i-th limb has the following relation:

$${}^o\dot{\boldsymbol{x}}_i = {}^o\boldsymbol{J}_i\dot{\boldsymbol{q}}_i \tag{14}$$

where ${}^o\dot{\boldsymbol{x}}_i$ is the velocity of the end point, $\dot{\boldsymbol{q}}_i$ is the joint velocity, and ${}^o\boldsymbol{J}_i$ is the usual Jacobian matrix. The leading superscript o implies that the elements are represented on the body center coordinate system shown in Fig. 1, which is fixed on a humanoid robot.

In our case, the body center is floating, and thus the end point motion about the world coordinate system is written as follows:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{X}_i^{-1}\dot{\boldsymbol{x}}_o + \boldsymbol{X}_o{}^o\boldsymbol{J}_i\dot{\boldsymbol{q}}_i \tag{15}$$

where $\dot{\boldsymbol{x}}_o = [\dot{\boldsymbol{r}}_o^T; \boldsymbol{\omega}_o{}^T]^T$ is the velocity of the body center represented on the world coordinate system, and

$$\boldsymbol{X}_i = \begin{bmatrix} \boldsymbol{I}_3 & [\boldsymbol{R}_o{}^o\boldsymbol{r}_i\times] \\ \boldsymbol{0}_3 & \boldsymbol{I}_3 \end{bmatrix} \tag{16}$$

is a (6×6) matrix which relates the body center velocity and the i-th limb velocity. $\boldsymbol{I}_3$ and $\boldsymbol{0}_3$ are an (3×3) identity and zero matrix, respectively. $\boldsymbol{R}_o{}^o\boldsymbol{r}_i$ is the position vector from the body center to the end point of the i-th limb represented on the world coordinate frame. $[(\cdot)\times]$ is a skew-symmetric matrix for the cross product. The transformation matrix $\boldsymbol{X}_o$ is

$$\boldsymbol{X}_o = \begin{bmatrix} \boldsymbol{R}_o & \boldsymbol{0}_3 \\ \boldsymbol{0}_3 & \boldsymbol{R}_o \end{bmatrix} \tag{17}$$

where $\boldsymbol{R}_o$ is the orientation of the body center represented on the world coordinate frame, and hereafter, we will use the relation $\boldsymbol{J}_i = \boldsymbol{X}_o{}^o\boldsymbol{J}_i$.

From Eq. (15), we can see that all the limbs should satisfy the compatibility condition that the body center velocity is the same, and thus i-th limb and j-th limb should satisfy the following relation:

$$\dot{\boldsymbol{x}}_o = \boldsymbol{X}_i(\dot{\boldsymbol{x}}_i - \boldsymbol{J}_i\dot{\boldsymbol{q}}_i) = \boldsymbol{X}_j(\dot{\boldsymbol{x}}_j - \boldsymbol{J}_j\dot{\boldsymbol{q}}_j). \tag{18}$$

From Eq. (18), the joint velocity of any limb can be represented by the joint velocity of the base limb and desired cartesian motions of limbs. Actually, the base limb should be chosen to be the supporting leg in the single supporting phase or one of both legs in the double supporting phase. Let us express the base limb with the subscript 1, then the joint velocity of any limb is expressed as:

$$\dot{\boldsymbol{q}}_i = \boldsymbol{J}_i^{-1}\dot{\boldsymbol{x}}_i - \boldsymbol{J}_i^{-1}\boldsymbol{X}_{i1}(\dot{\boldsymbol{x}}_1 - \boldsymbol{J}_1\dot{\boldsymbol{q}}_1), \tag{19}$$

for $i = 2, \cdots, n$. Here,

$$\boldsymbol{X}_{i1} \triangleq \boldsymbol{X}_i^{-1}\boldsymbol{X}_1 = \begin{bmatrix} \boldsymbol{I}_3 & [\boldsymbol{R}_o({}^o\boldsymbol{r}_1 - {}^o\boldsymbol{r}_i)\times] \\ \boldsymbol{0}_3 & \boldsymbol{I}_3 \end{bmatrix}. \tag{20}$$

Note that if a limb is a redundant system, any null space optimization scheme can be added in Eq. (19). Now, let us rewrite the conventional CoM Jacobian explained in [9] as follows:

$$\dot{\boldsymbol{c}} = \dot{\boldsymbol{r}}_o + \boldsymbol{\omega}_o \times (\boldsymbol{c} - \boldsymbol{r}_o) + \sum_{i=1}^{n} \boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\dot{\boldsymbol{q}}_i. \tag{21}$$

where n is the number of limbs, $\boldsymbol{c}$ is the position vector of CoM represented on the world coordinate system, namely, $\boldsymbol{c} = [c_x, c_y, c_z]^T$, and ${}^o\boldsymbol{J}_{c_i}$ means CoM Jacobian of i-th limb represented on the body center coordinate frame. Here, the motion of body center frame can be obtained by using Eq. (15) for the base limb as follows:

$$\begin{aligned} \dot{\boldsymbol{x}}_o &= \boldsymbol{X}_1\left\{\dot{\boldsymbol{x}}_1 - \boldsymbol{X}_o{}^o\boldsymbol{J}_1\dot{\boldsymbol{q}}_1\right\} \\ \begin{bmatrix}\dot{\boldsymbol{r}}_o \\ \boldsymbol{\omega}_o\end{bmatrix} &= \begin{bmatrix} \boldsymbol{I}_3 & [\boldsymbol{R}_o{}^o\boldsymbol{r}_1\times] \\ \boldsymbol{0}_3 & \boldsymbol{I}_3\end{bmatrix}\left\{\begin{bmatrix}\dot{\boldsymbol{r}}_1 \\ \boldsymbol{\omega}_1\end{bmatrix} - \begin{bmatrix}\boldsymbol{R}_o{}^o\boldsymbol{J}_{v_1} \\ \boldsymbol{R}_o{}^o\boldsymbol{J}_{\omega_1}\end{bmatrix}\dot{\boldsymbol{q}}_1\right\}, \end{aligned} \tag{22}$$

where ${}^o\boldsymbol{J}_{v_1}$ and ${}^o\boldsymbol{J}_{\omega_1}$ are the linear and angular velocity part of the base limb Jacobian. Now, if Eq. (19) for other limbs is applied to Eq. (21), the CoM motion is rearranged as follows:

$$\begin{aligned} \dot{\boldsymbol{c}} = &\ \dot{\boldsymbol{r}}_o + \boldsymbol{\omega}_o \times (\boldsymbol{c} - \boldsymbol{r}_o) + \boldsymbol{R}_o{}^o\boldsymbol{J}_{c_1}\dot{\boldsymbol{q}}_1 \\ &+ \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}(\dot{\boldsymbol{x}}_i - \boldsymbol{X}_{i1}\dot{\boldsymbol{x}}_1) + \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}\boldsymbol{X}_{i1}\boldsymbol{J}_1\dot{\boldsymbol{q}}_1. \end{aligned}$$

Here, if Eq. (22) is applied to above equation, then the CoM motion is only related to the base limb:

$$\begin{aligned} \dot{\boldsymbol{c}} = &\ \dot{\boldsymbol{r}}_1 + \boldsymbol{\omega}_1 \times \boldsymbol{r}_{c1} - \boldsymbol{R}_o{}^o\boldsymbol{J}_{v_1}\dot{\boldsymbol{q}}_1 + \boldsymbol{r}_{c1} \times \boldsymbol{R}_o{}^o\boldsymbol{J}_{\omega_1}\dot{\boldsymbol{q}}_1 \\ &+ \boldsymbol{R}_o{}^o\boldsymbol{J}_{c_1}\dot{\boldsymbol{q}}_1 + \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}(\dot{\boldsymbol{x}}_i - \boldsymbol{X}_{i1}\dot{\boldsymbol{x}}_1) \\ &+ \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}\boldsymbol{X}_{i1}\boldsymbol{J}_1\dot{\boldsymbol{q}}_1 \end{aligned} \tag{23}$$

where $\boldsymbol{r}_{c1} = \boldsymbol{c} - \boldsymbol{r}_1$. Also, if the base limb is stuck to the ground ($\dot{\boldsymbol{r}}_1 = \boldsymbol{0}$ and $\boldsymbol{\omega}_1 = \boldsymbol{0}$), then Eq. (23) is simplified as follows:

$$\begin{aligned} \dot{\boldsymbol{c}} = &\ -\boldsymbol{R}_o{}^o\boldsymbol{J}_{v_1}\dot{\boldsymbol{q}}_1 + \boldsymbol{r}_{c1} \times \boldsymbol{R}_o{}^o\boldsymbol{J}_{\omega_1}\dot{\boldsymbol{q}}_1 + \boldsymbol{R}_o{}^o\boldsymbol{J}_{c_1}\dot{\boldsymbol{q}}_1 \qquad (24) \\ &+ \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}(\dot{\boldsymbol{x}}_i - \boldsymbol{X}_{i1}\dot{\boldsymbol{x}}_1) + \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}\boldsymbol{X}_{i1}\boldsymbol{J}_1\dot{\boldsymbol{q}}_1. \end{aligned}$$

Finally, all the given desired limb motions, $\dot{\boldsymbol{x}}_i$ are embedded in the CoM Jacobian. Thus the effect of the CoM movement generated by the given limb motion is compensated by the base limb. Eq. (24) can be rewritten like the usual kinematic Jacobian of base limb:

$$\dot{\boldsymbol{c}}_{\text{emc}} = \boldsymbol{J}_{\text{emc}}\dot{\boldsymbol{q}}_1 \tag{25}$$

where

$$\dot{\boldsymbol{c}}_{\text{emc}} = \dot{\boldsymbol{c}} - \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}(\dot{\boldsymbol{x}}_i - \boldsymbol{X}_{i1}\dot{\boldsymbol{x}}_1), \tag{26}$$

$$\begin{aligned} \boldsymbol{J}_{\text{emc}} = &\ \boldsymbol{R}_o\left[-{}^o\boldsymbol{J}_{v_1} + \boldsymbol{r}_{c1} \times {}^o\boldsymbol{J}_{\omega_1} + {}^o\boldsymbol{J}_{c_1}\right] \\ &+ \sum_{i=2}^{n}\boldsymbol{R}_o{}^o\boldsymbol{J}_{c_i}\boldsymbol{J}_i^{-1}\boldsymbol{X}_{i1}\boldsymbol{J}_1. \end{aligned} \tag{27}$$

The CoM motion with embedded limb motions, $\dot{\boldsymbol{c}}_{\text{emc}}$, consists of two relations: a given desired CoM motion(the first term) which is derived in the previous section, and the relative effect of other limbs(the second term). The CoM Jacobian with embedded limb motions, $\boldsymbol{J}_{\text{emc}}$ also consists of three relations: the effect of the body center(the first and the second term), the effect of the base limb(the third term), and the effect of other limbs(the last term).

The CoM Jacobian with embedded motion $\boldsymbol{J}_{\text{emc}}$ is a $(3 \times n_1)$ matrix where n_1 is the dimension of the base limb, which is smaller than that of the original CoM Jacobian, thus the calculation time can be reduced. After solving Eq. (25), the joint motion of the base limb is obtained. The resulting base limb motion makes a humanoid robot balanced automatically during the movement of the all other limbs. With the joint motion of the base limb, the joint motion of all the other limbs are obtained by Eq. (19). The resulting motion follows the given desired motion, regardless of the balancing motion of the base limb. In other words, the suggested kinematic resolution method offers the WBC(whole body coordination) function to the humanoid robot automatically.

V. Stability of Walking Controller

Since a bipedal walking robot is an electro-mechanical system including many electric motors, gears and link mechanisms, there exist many disturbances in executing the motions of the pre-generated desired trajectories of CoM and ZMP for a real bipedal robot system. To show the robustness of the controller against disturbances, we apply the following stability theory to a bipedal robot control system. The control system is said to be disturbance input-to-state stable (ISS) [11], if there exists a smooth positive definite radially unbounded function $V(\boldsymbol{e}, t)$, a class $\mathcal{K}_\infty$ function γ_1 and a class $\mathcal{K}$ function γ_2 such that the following dissipativity inequality is satisfied:

$$\dot{V} \leq -\gamma_1(|\boldsymbol{e}|) + \gamma_2(|\boldsymbol{\epsilon}|), \tag{28}$$

where $\dot{V}$ represents the total derivative for Lyapunov function, $\boldsymbol{e}$ the error state vector and $\boldsymbol{\epsilon}$ disturbance input vector.

In this section, we propose the walking controller for bipedal robot systems as shown in Fig. 4. In this figure, first, the ZMP Planer and CoM Planer generate the desired trajectories in Fig. 3 which are satisfying the following differential equation:

$$p_i^d = c_i^d - 1/\omega_n^2\ddot{c}_i^d \quad \text{for} \quad i = x, y. \tag{29}$$

Second, the simplified model for the real bipedal walking robot has the following dynamics:

$$\begin{aligned} \dot{c}_i &= u_i + \epsilon_i \\ p_i &= c_i - 1/\omega_n^2\ddot{c}_i \quad \text{for} \quad i = x, y, \end{aligned} \tag{30}$$

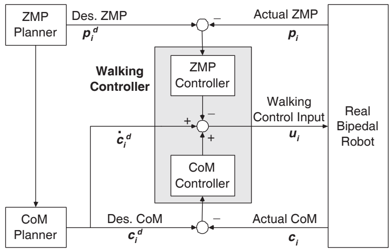

Fig. 4. Walking Controller for a Bipedal Walking Robot

where ϵ_i is the disturbance input produced by actual control error, u_i is the walking control input, c_i and p_i are the actual CoM and ZMP positions of the real bipedal robot, respectively. The real bipedal robot makes the kinematic resolution from the walking control input to the motor driving joint velocity as explained in the previous section. Concretely speaking, the walking control input is applied to the term $\dot{\boldsymbol{c}}$ in Eq. (26) by replacing $\dot{c}_i$ with u_i, for $i = x, y$. Also, the real bipedal robot offers the ZMP information from force/torque sensors attached to the ankles of humnaoid and the CoM information from the encoder data attached to the motor driving axes, respectively, as shown in Fig. 4. Here, we assume that the the disturbance produced by control error is bounded and its differentiation is also bounded, namely, $|\epsilon_i| < a$ and $|\dot{\epsilon}_i| < b$ with positive constants a and b. Also, we should notice that the control error always exists in real robot systems and its magnitude depends on the performance of embedded local servos. The following theorem proves the stability of the walking controller for the simplified walking robot model.

Theorem 1: Let us define the ZMP and CoM error for the simplified bipedal walking robot control system (30) as follows:

$$\begin{aligned} e_{p,i} &\triangleq p_i^d - p_i \\ e_{c,i} &\triangleq c_i^d - c_i, \quad \text{for} \quad i = x, y. \end{aligned}$$

If the walking control input u_i in Fig. 4 has the following form:

$$u_i = \dot{c}_i^d - k_{p,i} e_{p,i} + k_{c,i} e_{c,i} \tag{31}$$

under the gain conditions:

$$k_{c,i} > \omega_n \quad \text{and} \quad 0 < k_{p,i} < \left(\frac{\omega_n^2 - \beta^2}{\omega_n} - \gamma^2 \right) \tag{32}$$

with the positive arbitrary numbers satisfying the following conditions:

$$\beta < \omega_n \quad \text{and} \quad \gamma < \sqrt{\frac{\omega_n^2 - \beta^2}{\omega_n}},$$

then the walking controller gives the disturbance input($\epsilon_i, \dot{\epsilon}_i$)-to-state($e_{p,i}, e_{c,i}$) stability (ISS) to a simplified bipedal walking robot, where, the $k_{p,i}$ is the proportional gain of ZMP controller and $k_{c,i}$ is that of CoM controller in Fig. 4.

Proof. First, we get the error dynamics from Eq. (29) and (30) as follows:

$$\ddot{e}_{c,i} = \omega_n^2 (e_{c,i} - e_{p,i}). \tag{33}$$

Second, another error dynamics is obtained by using Eq. (30) and (31) as follows:

$$k_{p,i} e_{p,i} = \dot{e}_{c,i} + k_{c,i} e_{c,i} + \epsilon_i, \tag{34}$$

also, this equation can be rearranged for $\dot{e}_c$:

$$\dot{e}_{c,i} = k_{p,i} e_{p,i} - k_{c,i} e_{c,i} - \epsilon_i. \tag{35}$$

Third, by differentiating the equation (34) and by using equations (33) and (35), we get the following:

$$\begin{aligned} \dot{e}_{p,i} &= 1/k_{p,i} \left(\ddot{e}_{c,i} + k_{c,i} \dot{e}_{c,i} + \dot{\epsilon}_i \right) \\ &= \omega_n^2 / k_{p,i} (e_{c,i} - e_{p,i}) \\ &\quad + k_{c,i}/k_{p,i} (k_{p,i} e_{p,i} - k_{c,i} e_{c,i} - \epsilon_i) + (1/k_{p,i}) \dot{\epsilon}_i \\ &= \left(\frac{\omega_n^2 - k_{c,i}^2}{k_{p,i}} \right) e_{c,i} - \left(\frac{\omega_n^2 - k_{p,i} k_{c,i}}{k_{p,i}} \right) e_{p,i} \\ &\quad + \frac{1}{k_{p,i}} (\dot{\epsilon}_i - k_{c,i} \epsilon_i). \end{aligned} \tag{36}$$

Fourth, let us consider the following Lyapunov function:

$$V(e_{c,i}, e_{p,i}) \triangleq \frac{1}{2} \left[(k_{c,i}^2 - \omega_n^2) e_{c,i}^2 + k_{p,i}^2 e_{p,i}^2 \right], \tag{37}$$

where $V(e_c, e_p)$ is the positive definite function for $k_{p,i} > 0$ and $k_{c,i} > \omega_n$, except $e_{c,i} = 0$ and $e_{p,i} = 0$. Now, let us differentiate the above Lyapunov function

$$\begin{aligned} \dot{V} &= (k_{c,i}^2 - \omega_n^2) e_{c,i} \dot{e}_{c,i} + k_{p,i}^2 e_{p,i} \dot{e}_{p,i} \\ &= -k_{c,i} (k_{c,i}^2 - \omega_n^2) e_{c,i}^2 - k_{p,i} (\omega_n^2 - k_{p,i} k_{c,i}) e_{p,i}^2 \\ &\quad - (k_{c,i}^2 - \omega_n^2) e_{c,i} \epsilon_i + k_{p,i} e_{p,i} \dot{\epsilon}_i - k_{p,i} k_{c,i} e_{p,i} \epsilon_i \\ &= -(k_{c,i} - \alpha^2)(k_{c,i}^2 - \omega_n^2) e_{c,i}^2 \\ &\quad - k_{p,i} [\omega_n^2 - (k_{p,i} + \gamma^2) k_{c,i} - \beta^2] e_{p,i}^2 \\ &\quad - (k_{c,i}^2 - \omega_n^2) \left| \alpha e_{c,i} + \frac{1}{2\alpha} \epsilon_i \right|^2 - k_{p,i} \left| \beta e_{p,i} - \frac{1}{2\beta} \dot{\epsilon}_i \right|^2 \\ &\quad - k_{p,i} k_{c,i} \left| \gamma e_{p,i} + \frac{1}{2\gamma} \epsilon_i \right|^2 \\ &\quad + \left[\frac{(k_{c,i}^2 - \omega_n^2)}{4\alpha^2} + \frac{k_{p,i} k_{c,i}}{4\gamma^2} \right] \epsilon_i^2 + \frac{k_{p,i}}{4\beta^2} \dot{\epsilon}_i^2 \end{aligned}$$

Therefore,

$$\begin{aligned} \dot{V} \leq\; & -(k_{c,i} - \alpha^2)(k_{c,i}^2 - \omega_n^2) e_{c,i}^2 \\ & - k_{p,i} [\omega_n^2 - (k_{p,i} + \gamma^2) k_{c,i} - \beta^2] e_{p,i}^2 \\ & + \left[\frac{(k_{c,i}^2 - \omega_n^2)}{4\alpha^2} + \frac{k_{p,i} k_{c,i}}{4\gamma^2} \right] \epsilon_i^2 + \frac{k_{p,i}}{4\beta^2} \dot{\epsilon}_i^2 \end{aligned} \tag{38}$$

where $e_{c,i}^2$ term is negative definite with the arbitrary positive number satisfying $\alpha < \sqrt{\omega_n}$ and $e_{p,i}^2$ term is negative definite under the given conditions (32). Here, since the inequality (38) follows the ISS property (28), we concludes that the proposed walking controller gives the disturbance input($\epsilon_i, \dot{\epsilon}_i$)-to-state($e_{p,i}, e_{c,i}$) stability (ISS) to the simplified control system model of bipedal walking robot. □

Remark 1: Note that the ZMP controller in above theorem has the negative feedback different from the conventional controller. Also, for practical use, the gain conditions of walking controller can be simply rewritten without arbitrary positive numbers β and γ as follows:

$$k_{c,i} > \omega_n \quad \text{and} \quad 0 < k_{p,i} < \omega_n,$$

because the stability proof is very conservative in above theorem.

VI. Experimental Results

First, we experimented the humanoid robot dancing to show the WBC(whole body coordination) function of the kinematic resolution method developed in section IV. The desired dancing motion shown in Fig. 5 is applied to the dual arms, then the supporting (base) limb motion is generated from the kinematic resolution method of CoM Jacobian with embedded dancing motion. The initial positions of CoM are $c_x = 0.034[m]$, $c_y = 0.0[m]$, $c_z = 0.687[m]$, respectively. Though the joint configurations of dual arms are changed with the dancing motion as shown in Fig. 5, the position of CoM is not nearly changed at the initial position as shown in Fig. 6. Also, we can see in Fig. 6 that the ZMP has the small changes within the bounds of $\pm 0.01[m]$ approximately. As a result, we could succeed in implementing the fast dancing motion stably thanks to the WBC function.

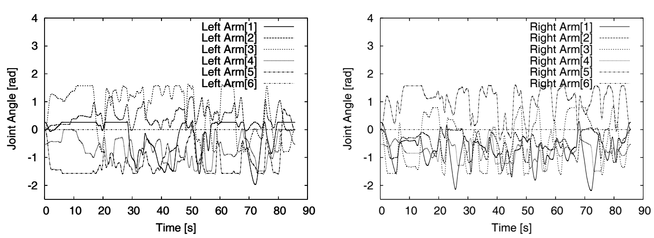

Fig. 5. Experimental Result : Joint Trajectories of Arms while Dancing

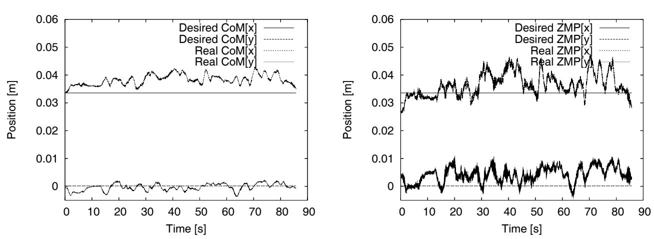

Fig. 6. Experimental Result : CoM and ZMP Trajectories while Dancing

Second, in order to demonstrate the effectiveness of walking controller proposed in section V, the desired ZMP/CoM trajectories are generated by setting $T = 1.0[s]$, $t_d = 0.1[s]$, $A = 0.09[m]$, $B = 0.1[m]$, $m = 67.68[kg]$, $c_z = 0.687[m]$, and $\omega_n = \sqrt{g/c_z} = 3.78$, and the gains of walking controller are set as $k_{p,i} = \{3.0, 1.8\}$, and $k_{c,i} = \{6.6, 3.8\}$ for $i = x, y$. And then, we experimented the proposed controller with humanoid robot. The experimental results are shown in Fig. 7. These results demonstrate the stability of the proposed walking controller while following the desired CoM and ZMP trajectories.

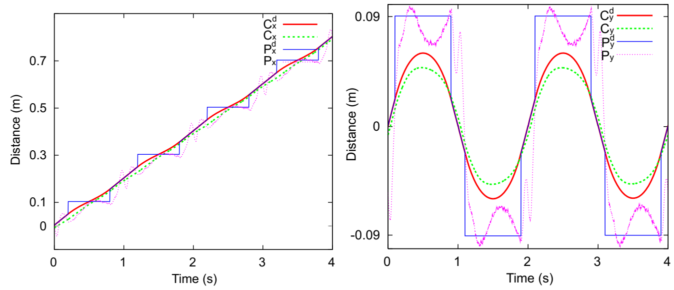

Fig. 7. Experimental Result : CoM and ZMP Trajectories while walking, where c_i means the actual position of CoM, c_i^d the desired position of CoM, p_i the actual ZMP, and p_i^d the desired ZMP for $i = x, y$.

VII. Concluding Remarks

In this paper, the desired CoM/ZMP trajectory planning method, the kinematic resolution method of CoM Jacobian with an embedded walking or dancing motion, and the walking control method were proposed for the humanoid robot. The proposed kinematic resolution method with CoM Jacobian offers the whole body coordination function to the humanoid robot automatically. Also, The disturbance input-to-state stability (ISS) of the proposed walking controller was proved to show the robustness against disturbances. Finally, we showed the effectiveness of the proposed methods through the experiments.

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