IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 6, NO. 1, FEBRUARY 1990

Gait Synthesis for the SD-2 Biped Robot to Climb Sloping Surface

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Abstract—**A scheme to enable the SD-2 biped robot to climb sloping surfaces is proposed. By means of sensing devices, namely position sensors on the joints and force sensors underneath the heel and toe, a biped robot called SD-2 is able to detect the transition of the supporting terrain from a flat floor to a sloping surface. An algorithm is developed for the biped robot control system to evaluate the inclination of the supporting foot and the unknown gradient, and a compliant motion scheme is then used to enable the robot to transfer from level walking to climbing the slope. While the robot walks on the slope, the gait synthesis is a simple modification to the one used for level walking. Experiments with the SD-2 biped robot show that the overall scheme, while simple to implement, is powerful and reliable enough to permit walking from level to slope or *vice versa*. Finally, it is argued that the mechanism proposed in this paper can be extended to quasi-dynamic and dynamic gaits.**

I. INTRODUCTION

RESEARCH on biped robots and biped locomotion has been conducted for a number of years. While theoretical studies were started as early as twenty years ago by many scientists [1]–[7], practical biped robots have been relatively scarce. In 1973 the first biped walking in the world [8] was accomplished by an eleven-degrees-of-freedom biped robot WL-5, designed and constructed by Kato and his colleagues. Its gait was static walking, i.e., the robot center-of-gravity was kept above at least one of its large feet. Later Kato and his coworkers developed a locomotion gait which was called quasi-dynamic walking [9]. This was regarded as a transitional step from static to dynamic walking. Recently, dynamic walking was realized for a biped robot called WL-10RD, which was built by the same group of people [10].

In addition to Kato's group, many other scientists have designed and constructed various kinds of biped robots such as a dynamic walking robot by Miura and Shimoyama [11], a similar one by Raibert [12], a seven-degrees-of-freedom biped robot by Arimoto and Miyazaki [13], and more recently a biped robot called SD-2 by Zheng [14].

In all the above research projects, biped walking was confined to a flat floor. The problem of biped locomotion on more difficult terrain has never been addressed. Actually, this problem is of significant importance since one of the main reasons to explore the use of legs for locomotion is the need for vehicles that can travel in difficult terrain [15]. As stable walking on a flat floor has long been realized by a number of biped robots, we feel that it is time to study biped locomotion on uneven terrain. In this paper, a control scheme to permit a biped robot to walk from a level to a sloping surface is proposed (Fig. 1). The sloping surface may have an unpredictable gradient in the direction of walking. But both the level and slope do not have inclination in the frontal plane of the biped. Here, the frontal plane is perpendicular to the direction of walking. Later, we will explain that the same control scheme is suitable for more general concave terrains.

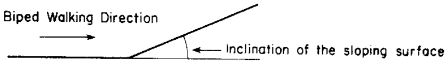

Fig. 1. A level followed by a sloping surface.

Walking on a surface that includes a transition from level to a slope is not as difficult as walking on irregular rough terrain. However, a biped robot will face the same problem of adapting itself to the alternation of the ground surface from level to slope or *vice versa*. The biped should at least be able, like a blind person, to give a proper response when the regular gait is suddenly obstructed by a sloping surface. This includes using its sensory system to measure the gradient of the slope and planning a new gait in order to continue its stable locomotion on the slope.

A static motion strategy is proposed in this paper for a biped robot to walk from a level to a sloping surface. The strategy includes three aspects. The first aspect is the scheme for detecting and measuring the gradient of the slope. The second aspect deals with the gait for the robot to walk on the slope. The last and most difficult aspect is the scheme for the robot to walk through the transition area joining the level ground and the slope.

The SD-2 biped robot constructed at Clemson University will be used as the model for this study. It has nine links and eight joints as shown in Fig. 2. Four joints are used to control the motion in the sagittal (or fore-and-aft) plane and the other four for the frontal plane (Fig. 2(b)). Each leg has four degrees of freedom. The top two joints emulate the hip joint while the bottom two are for the ankle joint. Note that the robot does not have knee joints. To enhance its adaptive capability, a pair of force sensors are mounted beneath the heel and toe of each foot (Fig. 3). Since in the frontal plane almost no difference exists for biped locomotion on level or on slope, our study will be concentrated on the sagittal plane.

Manuscript received April 20, 1988; revised July 27, 1989. This work was supported by the National Science Foundation under Contracts DMC-8620409, DMC-8657416, and by the Savannah River Laboratory under Contract AX-0721012. Part of the material in this paper was presented at the 1988 IEEE International Conference on Robotics and Automation, Philadelphia, PA, April 24–29, 1988.

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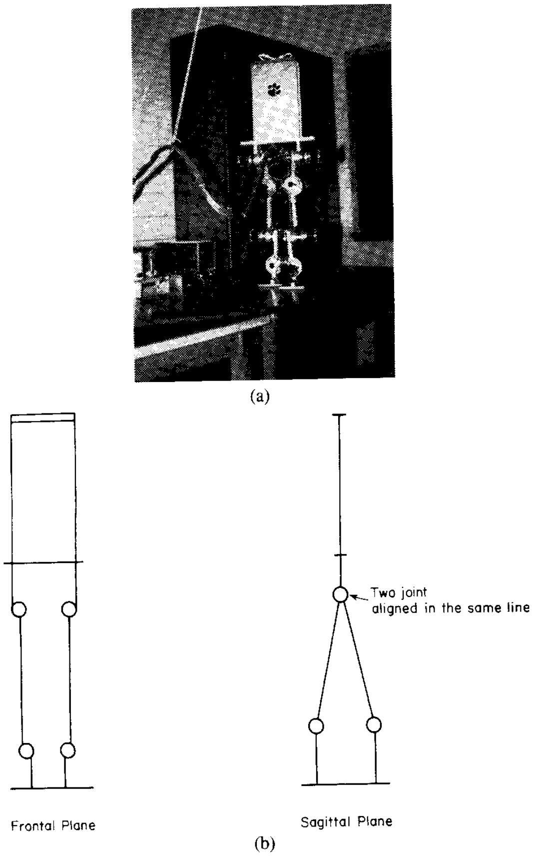
IEEE Log Number 8931964.



(a)

(b)

Fig. 2. (a) SD-2 biped robot. (b) Structure of the SD-2 biped robot in the frontal and sagittal planes.

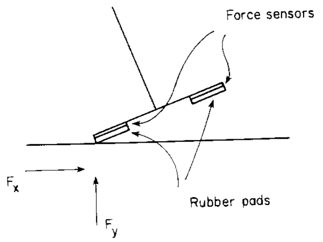
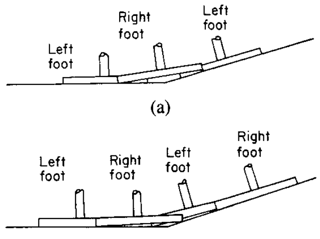


Fig. 3. Installation of the force sensors.

The structure of the paper is as follows. In the next section the method for detecting a sloping surface and measuring its gradient will be presented. In the third section a scheme for biped walking on the sloping surface will be developed. The fourth section will be devoted to a discussion of the motion mechanism for making the transition from walking on level ground to walking on a sloping surface. Finally, experiments on the SD-2 biped robot using the proposed scheme will be described in the fifth section, followed by the conclusions of this paper.

II. Measuring of the Inclination of Foot and the Gradient of Slope

For the SD-2 robot to climb a slope from a flat floor, one of two types of transition must be made as shown in Fig. 4. The transition shown in Fig. 4(a) includes a state when one foot is bridging the joint between the flat floor and the slope. The other foot starts by resting on the flat floor and then moves to a position completely on the sloping surface. A second possible transition shown in Fig. 4(b) involves an intermediate state when *both* feet are resting partially on the flat floor and partially on the slope. Neither alternative is predictable.



(a)

(b)

Fig. 4. Two possible transitions for the robot to climb a slope.

Each type of robot gait transition from flat floor to slope involves some or all of the three phases shown in Fig. 5.

Phase 1 occurs at the moment when the biped touches the slope for the first time. Phase 3 represents the moment that one of the biped feet completely stands on the slope. Phase 2 is a transient phase from Phase 1 to Phase 3. Both feet in this phase stand between the level and slope. Depending on how the biped approaches the slope, Phase 2 may or may not be included in the transition, which makes the difference between the first alternative without Phase 2 and the second alternative with Phase 2.

Since the center-of-gravity of a biped robot is always supported by its feet for static walking, it is of critical importance to detect the inclination of the newly landing foot. Here the inclination of a foot is defined as the angle between the foot and the ground which is less than 90°. We assume that the toe and heel sensors can detect the moment that both the heel and toe of a landing foot touch the ground. Then the inclination of the newly landing foot can be indirectly calculated by using joint positions. The methods of calculation will be discussed for all three phases.

Phase 1: We notice from Fig. 5 that the feet and legs form a hexagon in Phase 1. Let the angle of inclination for the front foot be denoted α. Then α can be expressed as

$$\alpha = q_1 + (q_2 + q_3) + q_4 - 4\pi \tag{1}$$

where q_1, q_2, q_3, and q_4 are robot joint angles that can be sensed by joint position sensors.

Phases 2 and 3: As it can be seen from Fig. 5, the inclination of the front foot α in these two phases can be expressed as

$$\alpha = \alpha_1 + \alpha_2 \tag{2}$$

where α_1 is the inclination of the back foot which was calculated in Phase 1 and α_2 is the included angle between the two feet which is less than 90°. If α_1 is zero degrees, α_2 is actually the inclination of the front foot. Therefore, we may

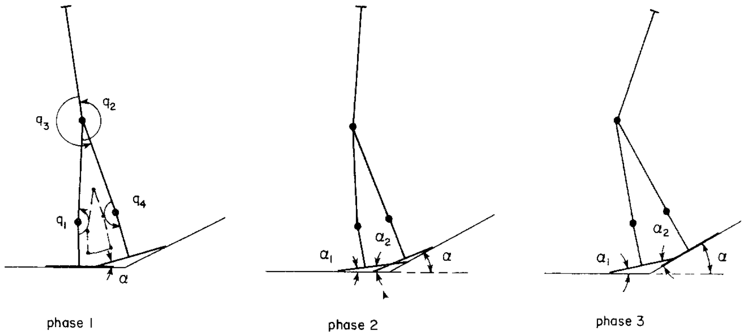


Fig. 5. Three phases of the robot gait transition from the level ground to a slope. Note that the dotted outline in phase 1 indicates that the feet and legs form a hexagon.

use (1) to calculate α_2 with joint angles being measured at the moment of Phase 2 or 3. Considering the fact that α_1 is equal to zero for Phase 1, we can take (2) as a general formula for calculating the inclination of the front foot. Then the following claim is in order.

Claim 1: When an SD-2 type robot climbs a slope from a level, an inclination of the front foot occurs. The inclination is equal to the sum of the included angle between the two feet and the inclination of the back foot.

From Claim 1 or (2), we may find that the inclination of the front foot can be recursively calculated so long as the first inclination is calculated by (1).

We next need to find out how to measure the gradient of the slope. Here the gradient is defined as the included angle between the level and slope which is less than 90°. In fact, when the robot is in Phase 3 the inclination of the front foot is the same as the gradient. But the robot cannot judge if the front foot is in Phase 2 or Phase 3 even when Phase 3 has been reached. The robot needs to take one more step following Phase 3 to determine that the included angle α_2 is zero. It is then known that the robot stands completely on the slope and the inclination of the front foot represents the gradient of the slope.

III. WALKING ON A SLOPING SURFACE

Walking from a level to a slope can be divided into three sequential periods: 1) walking on the level ground, 2) transitional walking from the level of the slope, and 3) walking on the sloping surface.

Walking on the level ground has previously been realized for the SD-2 biped robot [14]. Walking in the second period, on the other hand, will partly utilize the mechanism that is used for walking on the sloping surface. We will therefore discuss the walking gait for the sloping surface first in this section followed by the study of transitional walking in the next section.

It turns out that the gait for walking on a sloping surface only involves a simple modification to the one for walking on the level ground. Hence, we need to give a brief description of the level-ground gait first in order to develop a gait for the slope.

A. Walking on the Level Ground

A static walking gait for the SD-2 biped robot is shown in Fig. 6, which involves five states. From state 1 to state 2, robot joints in the sagittal plane are fixed while those in the frontal plane are moved to transfer the projection of the center of gravity from the center of the supporting area formed by two feet to the region of the left foot and then the right foot can be lifted. In state 3, joints in the frontal plane are fixed, but those in the sagittal plane are rotated to move the center of gravity forward. In state 4, joints in both the frontal and sagittal planes move simultaneously to make the right foot touch the ground while the center of gravity remains over the left foot. From state 4 to state 5, all the joints are rotated to shift the center of gravity to the center of the two-foot supporting area again. The five states just described complete a cycle of taking a single step. The same procedure can be repeated to take more steps.

B. Walking on a Sloping Surface

If the gait of walking on the level ground is used without modification for walking on a sloping surface, the projection of the center of gravity will move toward the back of the supporting area. The distance of the movement depends on the gradient of the slope. In general, it reduces the stability margin since the projection of the center of gravity is now closer to the back boundary of the supporting area. If the slope is too steep, the robot will fall; therefore, some effective measure is needed to prevent the center of gravity from shifting backwards. If the shift in the center of gravity is prevented, the biped robot can walk on the slope as stably as on the level ground.

Since the SD-2 biped robot does not have knee joints, we can only move the main body of the robot to compensate for the backward shift of the center of gravity. In order to find the required motion of the main body, the following analysis is used.

Referring to Fig. 7(a), it is known that the vertical projection of the center of gravity on a level ground can be expressed as

$$S(t) = \frac{1}{M}\sum_{i=1}^{5} m_i S_i(t) \tag{3}$$

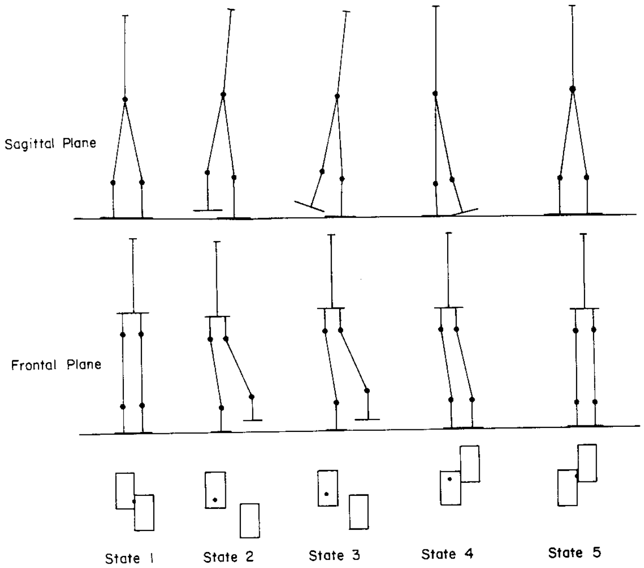
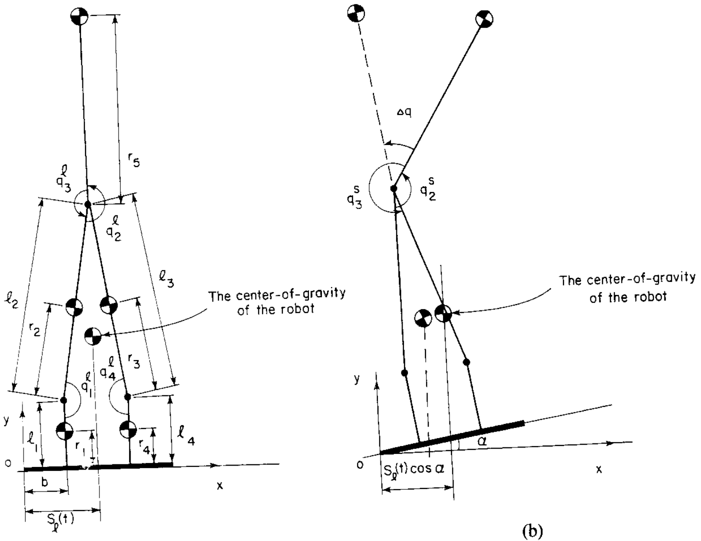


Fig. 6. Robot walking gait on the level ground.



(a) (b)

Fig. 7. Parameters of robot links.

where M is the total mass of the biped robot, m_i is the mass of the ith link (including the motor), and $S_i(t)$ is the projection of the center of gravity of the ith link.

Assume that the projection of the center of gravity of the robot is $S_l(t)$ when the robot is walking on the level (Fig. 7(a)). On the slope the projection will be less than $S_l(t)$ if exactly the level-ground walking pattern is used for walking on the slope (Fig. 7(b)). To maintain stability the robot needs to bend the main body forward such that the projection of the center of gravity will still go through the center of the supporting area to make the robot have the best stability margin. Assume that the inclination of the slope is α. Then the vertical projection of the center of gravity will be $S_l(t)\cos\alpha$.

To simplify the gait on the sloping surface we want the motions of the two legs to remain the same, with only the motion of the joints supporting the main body being modified. Let the joint positions be $q^l = [q_1^l \; q_2^l \; q_3^l \; q_4^l]^T$ when the robot walks on the level and be $q^s = [q_1^s \; q_2^s \; q_3^s \; q_4^s]^T$ when the robot walks on the slope. Then we should have (referring to Fig. 7(b)) $q_1^s = q_1^l$, $q_2^s = q_2^l - \Delta q$, $q_3^s = q_3^l + \Delta q$, and $q_4^s = q_4^l$. Consequently, we only need to determine Δq to design a stable walking gait for the slope. This may be done using (3).

When the robot is on the slope, each $S_i(t)$ on the right side of (3) can be found as

$$S_1(t) = b\cos\alpha - r_1 \sin\alpha$$

$$S_2(t) = b\cos\alpha - l_1 \sin\alpha + r_2 \sin(q_1^l + \alpha)$$

$$S_3(t) = b\cos\alpha - l_1 \sin\alpha + l_2 \sin(q_1^l + \alpha) - (l_3 - r_3)\sin(q_1^l + q_2^l + q_3^l + \alpha)$$

TABLE I
PARAMETERS OF THE SD-2 BIPED ROBOT

Link	Mass m_i (kg)	Radius r_i (m)	Radius l_i (m)	Foot Length b (m)
1	1.063	0.055	0.185	0.065
2	1.723	0.107	0.325	—
3	1.723	0.107	0.325	—
4	1.063	0.055	0.185	0.065
5	5.746	0.355	—	—

$$\begin{aligned} S_4(t) &= b\cos\alpha - l_1\sin\alpha + l_2\sin(q_1^l + \alpha) \\ &\quad - l_3\sin(q_1^l + q_2^l + q_3^l + \alpha) \\ &\quad - (l_4 - r_4)\sin(q_1^l + q_2^l + q_3^l + q_4^l + \alpha) \\ S_5(t) &= b\cos\alpha - l_1\sin\alpha + l_2\sin(q_1^l + \alpha) \\ &\quad - r_5\sin(q_1^l + q_2^l + \alpha - \Delta q) \end{aligned}$$

where b, l_i, and r_i are link parameters as shown in Fig. 7. Their values are summarized in Table I.

By substituting the above $S_i(t)$'s into (3), and letting $S(t) = S_l(t) \cdot \cos\alpha$, we may solve (3) for Δq, which is

$$\begin{aligned} \Delta q = -\sin^{-1}\Bigg[&\frac{1}{r_5 m_5}\{[-r_1 m_1 - l_1(M - m_1)]\sin\alpha \\ &+ [r_2 m_2 + l_2(m_3 + m_4 + m_5)]\sin(q_1^l + \alpha) \\ &- [(l_3 - r_3)m_3 + l_3 m_4]\sin(q_1^l + q_2^l + q_3^l + \alpha) \\ &- (l_4 - r_4)m_4\sin(q_1^l + q_2^l + q_3^l + q_4^l - \alpha) \\ &- M[S_l(t) - b]\cos\alpha\}\Bigg] + q_1^l + q_2^l + \alpha \end{aligned} \tag{4}$$

where $S_l(t)$ is the vertical projection of the center of gravity of the robot when it walks on the level. $S_l(t)$ can be expressed as

$$\begin{aligned} S_l(t) = \frac{1}{M}\{&Mb + [r_2 m_2 + l_2(m_3 + m_4 + m_5)]\sin q_1^l \\ &- [(l_3 - r_3)m_3 + l_3 m_4]\sin(q_1^l + q_2^l + q_3^l) \\ &- (l_4 - r_4)m_4\sin(q_1^l + q_2^l + q_3^l + q_4^l) \\ &- r_5 m_5\sin(q_1^l + q_2^l)\}. \end{aligned} \tag{5}$$

Equations (4) and (5) thus represent the modification to the walking gait for a flat floor which can be used to synthesize a gait for slope walking.

IV. TRANSITIONAL WALKING FROM LEVEL GROUND TO SLOPING SURFACE

The transitional gait used when walking from the level ground to the sloping surface involves continuously increasing the inclination of the landing foot. The normal walking gait on the level ground or on a sloping surface will not be suitable during the transition. In order to adapt to the changing environment, the robot must employ a "compliant" stepping motion. Here "compliant" means that the position and orientation of a foot when it lands on the ground are not pre-programmed but are determined by the force information generated by the force sensor at the moment of contact. As a result, the landing foot can comply with the constraints of the ground.

When walking on the level ground, the heel of the landing foot always touches the ground first. But when walking on the transitional area, either the heel or the toe may touch the ground first. The biped should manage the motion differently for the two touchings. If the heel touches the ground first, compliant motion is used for the landing foot to rotate about the heel until the toe touches the ground as well. Otherwise, the landing foot should rotate about the toe until the heel touches the ground. Since the instant of touching for both the heel and toe is unknown in transitional walking, we install force sensors underneath the heel and toe to detect the touching. The sensing signal will help the robot to accomplish the compliant motion. The complete gait of transitional walking is described as follows.

The gait for transitional walking is decomposed into a number of phases. Each phase will bring the robot from an old state to a new state as shown in Fig. 8. At state 1, the robot completely stands on the ground with the front foot having an inclination. We propose a virtual-slope-walking mechanism for the robot to take the next step, i.e., the front foot is assumed to be standing on the slope. Unfortunately, the back foot does not have the same inclination as the front foot; therefore, the slope walking gait developed in the previous section cannot be directly used. We need a *position adjusting phase* to turn state 1 into state 2 at which time the robot is in the status of slope walking. That is, the joint positions at state 2 belong to trajectories designed for slope walking with the inclination of the front foot as the gradient of the slope.

The position adjusting phase is important because it permits the robot to employ the virtual-slope-walking gait. Because state 1 does not belong to slope walking status, the regular slope walking gait cannot be applied. Only after state 2 is reached can the robot start to use the slope walking gait. The problem of turning state 1 into state 2 will be further addressed later in this section.

The virtual-slope-walking phase starts from state 2 and ends as soon as either of the following events occurs: 1) the heel of the new landing foot reaches the virtual slope (Fig. 8, state 3a); or 2) the toe of the new landing foot touches the real slope (Fig. 8, state 3b). If the first event occurs, then it will be followed by the *heel landing phase*; that is, state 3a to state 4 in which the heel moves down toward the slope until the real slope is touched. At state 3b or state 4, one part of the landing foot (either the toe or the heel) is in contact with the ground, from which the landing foot is brought to a complete contact with the ground which is state 5. We call the phase from state 4 to state 5 the *foot landing phase A*, and the one from state 3b to state 5 the *foot landing phase B*, respectively.

It should be noted that in the heel landing phase it is possible for the toe to touch the ground first before the landing foot reaches state 4, i.e., the landing foot goes from state 3a to state 3b instead of from state 3a to state 4. From state 3b, however, foot landing phase *B* will take over to bring the landing foot to state 5. From the above discussion, it can be seen that the following three combinations are possible for the gait of transitional walking:

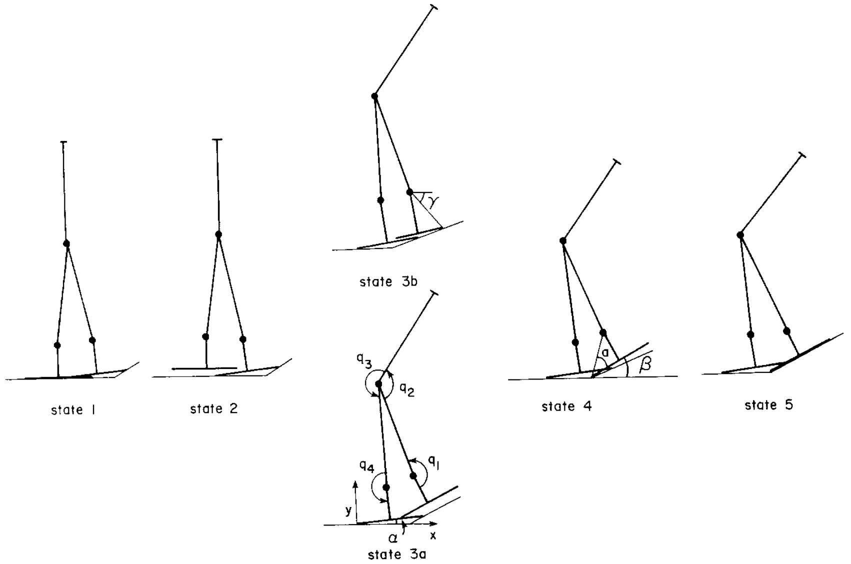
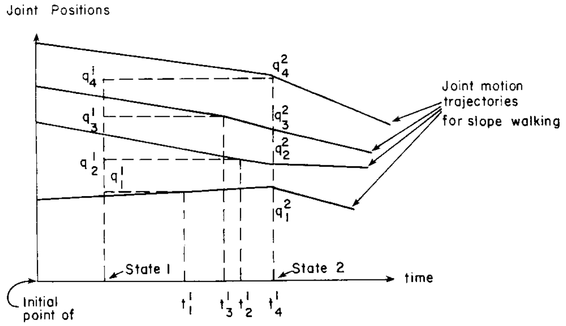


Fig. 8. States of transitional walking.



t_4^1 serves as the reference point of time since t_4^1 is greater than t_1^1, t_2^1, and t_3^1.

Fig. 9. Selection of joint positions for state 2 in the position adjusting phase.

a) state 1—state 2—state 3a—state 4—state 5,
b) state 1—state 2—state 3b—state 5,
c) state 1—state 2—state 3a—state 3b—state 5.

Among the above mentioned phases of transitional walking, only the motion in the virtual-slope-walking phase is well defined. The remaining phases, i.e., the position adjusting phase, the heel landing phase, and the foot landing phase including phase A and phase B, need to be carefully studied. They are individually studied as follows.

A. Position Adjusting Phase

The purpose of this phase is to move the robot from state 1 to state 2 such that the virtual-slope-walking mechanism can be used starting from state 2. At state 1, the joint positions $q^1 = [q_1^1 \ q_2^1 \ q_3^1 \ q_4^1]^T$ are not on the desired trajectories for slope walking (Fig. 9). However, at state 2, the joint positions $q^2 = [q_1^2 \ q_2^2 \ q_3^2 \ q_4^2]^T$ are on the desired trajectories for slope walking. The problem is how to select a reference point of time (the horizontal axis in Fig. 9) at which the joint positions of the desired trajectories can serve as state 2. We propose two criteria for selecting the joint positions of state 2. First, the joint positions at state 2 should be as close to state 1 as possible, which is called the shortest distance criterion. Secondly, the selected joint position for state 2 should result in forward motions for all the joints. Here "forward motions" means that the joint motions are in the process of lifting the back foot and making progress in taking a step rather than pushing the back foot further back to the ground. The obvious reason for the first criterion is to minimize the time of the position adjusting phase. The second criterion insures that the back foot will be lifted instead of being forced down against the ground, which would cause the whole robot to be lifted since the back foot is already on the ground.

To guarantee the forward motions as described above, we need to investigate the individual motion trajectories of slope walking. Assume that each joint trajectory in slope walking can be described as a function of time, i.e.,

$$q_i = f_i(t_i), \qquad i = 1, 2, 3, 4 \tag{6}$$

where t_i represents the elapsed time for joint i to reach q_i. From (6), we obtain

$$t_i = f_i^{-1}(q_i), \qquad i = 1, 2, 3, 4. \tag{7}$$

If the robot is in the status of slope walking, q_1, q_2, q_3, and q_4 should be resolved by (6) at one identical instant of time, i.e., $t_1 = t_2 = t_3 = t_4$. If the joint positions are not a proper combination of slope walking, using q_1, q_2, q_3, and q_4 in (7) will result in different t_i's.

Now we put the joint positions of state 1, q_i^1, ($i = 1, 2, 3, 4$), in (7) and obtain t_i^1, ($i = 1, 2, 3, 4$). Clearly, the t_i^1's are not the same. However, the t_i^1's tell us how far each joint position is from the initial state of slope walking (Fig. 9). If t_j^1 with $j = 1$, 2, 3, or 4 is the largest of all the t_i^1's, joint j has gone farther than any other joint on the desired trajectory of slope walking.

In order to have all the robot joints at the slope walking status at the same time and to have the forward motions as defined previously, the other joints have to catch up to joint j. Joint j, on the other hand, should not move until the other joints reach the desired positions of slope walking at the time t_j^1. Consequently, we select t_j^1 as the reference time. Then we use (6) and the reference time to calculate all the joint positions which will serve as state 2. Since joint j does not move during the catching up process, the other joints have the shortest distances to travel.

Fig. 9 further explains the selection procedure. In the figure, we find that t_4^1 is the greatest among all the t_i^1's. We then select t_4^1 as the reference time for state 2. The joint positions for state 2 are calculated as $q_1^2 = f_1(t_4^1)$, $q_2^2 = f_2(t_4^1)$, $q_3^2 = f_3(t_4^1)$, and $q_4^2 = f_4(t_4^1)$ which is the same as q_4^1. With these calculated positions, the robot moves from state 1 to state 2.

Once q^2 is determined, we use the following algorithm to plan the motion trajectories in phase 1:

$$q(t) = \frac{q^2 - q^1}{T} \cdot t + q^1 \tag{8}$$

where T is the duration of phase 1. In selecting T, we first find the joint position displacements between state 1 and state 2, i.e., $\Delta q_i = |q_i^2 - q_i^1|$, ($i = 1, 2, 3, 4$). We then calculate the time required for each joint to travel Δq_i, $T_i = \Delta q_i / \dot{q}_{im}$, where $\dot{q}_{im}$ is the maximum joint rate of joint i. The largest of T_i's is selected as T. This completes the motion planning of the robot in the position adjusting phase.

By the end of the position adjusting phase, the robot reaches state 2 which is traversed by a virtual-slope-walking phase. In this phase, the regular slope-walking gait is used until the heel reaches the virtual slope, i.e., state 3. If slope walking continues, the front foot will start to rotate. However, the heel has not touched the actual surface and the rotation of the foot will not guarantee a proper landing of the foot. In order for the heel to continue its downward motion, the robot starts the heel-landing phase from state 3. This is now explained.

B. Heel-Landing Phase

The goal in this phase is to make the landing foot move toward the slope in a direction perpendicular to the slope and without any rotating motion. The motion scheme is to keep the inclination of the landing foot unchanged until the heel touches the slope. It is possible, however, that the toe touches the ground before the heel. Since the robot motion in this phase is completely different from previously defined trajectories, new joint motion trajectories must be designed.

Referring to Fig. 8, state 3a, we propose to rotate joints 2, 3, and 4 to move the foot toward the slope and use the rotation of joint 1 to adjust the orientation of the landing foot. If we let the heel of the supporting foot be at the origin, the position of joint 1 can be expressed as

$$x = b\cos\alpha - l_4 \sin\alpha - l_3 \sin(q_4 - \alpha) + l_2 \sin(q_2 + q_3 + q_4 - \alpha) \tag{9}$$

and

$$y = b\sin\alpha + l_4 \cos\alpha - l_3 \cos(q_4 - \alpha) + l_2 \cos(q_2 + q_3 + q_4 - \alpha). \tag{10}$$

Next, take the derivative of (9) and (10) with respect to time to obtain

$$\dot{x} = -l_3 \cos(q_4 - \alpha)\dot{q}_4 + l_2 \cos(q_2 + q_3 + q_4 - \alpha) \cdot (\dot{q}_2 + \dot{q}_3 + \dot{q}_4) \tag{11}$$

and

$$\dot{y} = l_3 \sin(q_4 - \alpha)\dot{q}_4 - l_2 \sin(q_2 + q_3 + q_4 - \alpha) \cdot (\dot{q}_2 + \dot{q}_3 + \dot{q}_4). \tag{12}$$

The orientation of the landing foot at state 3a is $90^\circ + \alpha - q_1 - q_2 - q_3 - q_4$. Then to make joint 1 move perpendicular toward the slope it is necessary that

$$\dot{x}/\dot{y} = \tan\alpha. \tag{13}$$

From (11)–(13) one may obtain

$$[(A + B)\dot{q}_4 + B(\dot{q}_2 + \dot{q}_3)]/[-(C + D)\dot{q}_4 + D(\dot{q}_2 + \dot{q}_3)] = E \tag{14}$$

where

$$A = -l_3 \cos(q_4 - \alpha)$$
$$B = l_2 \cos(q_2 + q_3 + q_4 - \alpha)$$
$$C = l_3 \sin(q_4 - \alpha)$$
$$D = -l_2 \sin(q_2 + q_3 + q_4 - \alpha)$$

and

$$E = \tan\alpha.$$

Also it is necessary to make $\dot{q}_2 = \dot{q}_3$ such that the relative position of the main body with respect to the two legs is the same. As a result, the relation between $\dot{q}_4$ and $\dot{q}_2$ can be obtained from (14), i.e.,

$$\dot{q}_2 = -(A + B - EC - ED)\dot{q}_4 / 2(B + ED). \tag{15}$$

To keep the orientation of the landing foot constant, we see

$$\dot{q}_1 + \dot{q}_2 + \dot{q}_3 + \dot{q}_4 = 0 \tag{16}$$

which gives

$$\dot{q}_1 = -\dot{q}_2 - \dot{q}_3 - \dot{q}_4. \tag{17}$$

Thus (15), (17), and the relation $\dot{q}_3 = \dot{q}_2$ give the motion pattern for the heel landing phase. One may use joint 4 as an anchor joint and specify $\dot{q}_4$ to select the motion speed of the heel-landing phase.

It should be noted that the above motion mechanism will not adversely affect the stability problem. Since the landing foot is moving toward the slope, the supporting area will be increased in comparison to regular slope walking in which the landing foot moves toward the level ground. Furthermore, the relative motion of the main body with respect to the two legs is very limited since $\dot{q}_2 = \dot{q}_3$. This will prevent the position of the main body from affecting the center of gravity of the robot. The combination of the above two constraints keeps the robot in a statically stable status.

C. Foot-Landing Phase

1) Foot-Landing Phase A: Once the heel touches the ground, the foot-landing phase A starts. The landing foot must rotate about the heel until the toe touches the ground.

Assume that velocity of rotation is $\dot{\beta}$. Then the linear velocity of joint 1 can be expressed as

$$\dot{x} = \dot{\beta}\cos(\beta + a)\sqrt{l_1^2 + b^2} \tag{18}$$

and

$$\dot{y} = \dot{\beta}\sin(\beta + a)\sqrt{l_1^2 + b^2} \tag{19}$$

where $a = \arctan(l_1/b)$ and β is the inclination of the landing foot (Fig. 8, state 4) which can also be expressed as

$$\beta = 180^\circ + a - q_1 - q_2 - q_3 - q_4. \tag{20}$$

Since a is known and the joint positions can be measured, β can be calculated. The velocity of the foot rotation $\dot{\beta}$ can be selected by the robot operator, which will determine $\dot{x}$ and $\dot{y}$. With $\dot{x}$ and $\dot{y}$ being known and again letting $\dot{q}_2 = \dot{q}_3$, (11) and (12) can be used to solve for $\dot{q}_2$ and $\dot{q}_4$, as follows:

$$\dot{q}_2 = [(C + D)\dot{x} - (A + B)\dot{y}]/2[B(C + D) - D(A + B)] \tag{21}$$

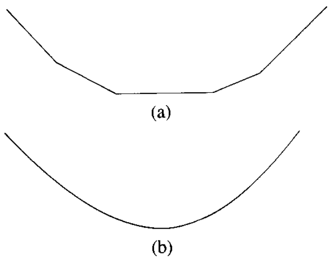
and

$$\dot{q}_4 = (D\dot{x} - B\dot{y})/[D(A + B) - B(C + D)] \tag{22}$$

where A, B, C, and D were previously defined. From (20) we may further obtain $\dot{q}_1$ which is

$$\dot{q}_1 = -\dot{\beta} - \dot{q}_2 - \dot{q}_3 - \dot{q}_4 = -\dot{\beta} - 2\dot{q}_2 - \dot{q}_4. \tag{23}$$

Thus (21)-(23) and the relation $\dot{q}_2 = \dot{q}_3$ define the motion mechanism for the foot-landing phase A.


(a)

(b)

Fig. 10. General concave surfaces.

2) Foot-Landing Phase B: Once the toe touches the ground, foot-landing phase B starts. The landing foot must rotate about the toe until the heel touches the ground.

Use γ to express the rotation angle of the foot. Here γ is the orientation of the line between joint 1 and the toe (Fig. 8, state 3b). It is given by

$$\gamma = 90^\circ + \alpha - q_1 - q_2 - q_3 - q_4 + \arctan\frac{b}{l_1}. \tag{24}$$

Then the linear velocity of joint 1 can be expressed as

$$\dot{x} = \dot{\gamma}\cos\gamma\sqrt{l_1^2 + b^2} \tag{25}$$

and

$$\dot{y} = \dot{\gamma}\sin\gamma\sqrt{l_1^2 + b^2} \tag{26}$$

where $\dot{\gamma}$ is the velocity of rotation selected by the robot operator. Once $\dot{x}$ and $\dot{y}$ are determined, we can use (21)-(23) again to find the joint velocities $\dot{q}_1$, $\dot{q}_2$, $\dot{q}_3$, and $\dot{q}_4$. But $\dot{\beta}$ in (21)-(23) should be replaced by $\dot{\gamma}$.

We have so far developed the transitional walking gait. The gait is suitable for the biped to walk from a level ground to a sloping surface whose gradient is unpredictable. It is clear that the same walking gait can be used for walking on the level ground or on the sloping surface. For the former, we may just consider that the inclination of the front foot is always zero, and thus the included angle is zero as well. For slope walking, once the bending angle of the body is determined, the robot may be treated as if it were walking on level ground since the rest of the joint motions remains the same. Consequently, the transitional walking gait is suitable for all the three kinds of surfaces. We therefore use the transitional walking gait for the biped if the terrain is unpredictable. However, if the terrain is predicted to be a level ground or a sloping surface for a large range, the gaits specially designed for the two cases can be used since they are computationally more efficient than the transitional walking gait.

Since walking on the slope surface may be treated as walking on the level ground, the transitional walking gait may be used for even more complicated terrain. First, it is suitable for the biped to walk down from a sloping surface to a level ground. It follows that the gait is valid if the robot walking on a concave surface formed by slopes (Fig. 10(a)) where both going up and down slopes are possible. Finally, the gait is even suitable for a smooth concave surface (Fig. 10(b)) if we consider the area underneath the supporting foot as a transitional area composed of different slopes.

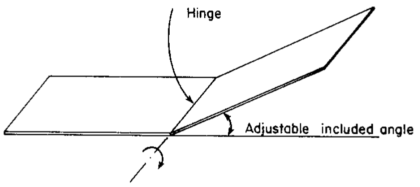


Fig. 11. Adjustable sloping surface for the experiment.

V. The Experiments

In this section, we will describe the experiments conducted in our laboratory and the experimental results. We applied the mechanism developed in the previous sections to the SD-2 robot and realized a successful concave surface walking. The detail of the experiment is described as follows.

A. The Hardware

In the experiment, the concave terrain was made up by a prepared surface which was constructed by hinging two flat boards (Fig. 11). The hinging between the boards made the included angle of the boards easily adjustable. The structure of the robot and the control system originally designed for level walking [14] have been modified to accomplish the experiment. The major modifications are as follows.

a) A 2 × 7.5 × 12.5 in^3 body was added to the biped robot. The body is a hollow aluminum box, but at the top of the body is a 2-lb steel block, which makes the center of gravity of the body almost at the top. Such a body enable the SD-2 biped robot to walk up or down a slope with up to 10° gradient while the body leans forward or backward by up to 70°, which is the limit of the body bending.

b) A pair of the force sensors were mounted beneath the heal and toe of each foot, more precisely, between the foot plate and its rubber pads (Fig. 3). The sensors used were called Force Sensing Resistors (FSR) which have been recently invented by Interlink Electronic [16]. The device changes resistance in a predictable manner when the surface is pressed.

c) A logic circuit which processed the force signal was developed and interfaced to the control computer as shown in Fig. 12. The force sensor was connected to the base of the transistor contained in the circuit (Fig. 12). Without force action on the sensor, the sensor had an infinitely large resistance which put the transistor in an "off" status. Once the sensor was pressed, the resistance instantly dropped to a small value which drove the transistor into an "on" status. As a result, the output voltage from the collector of the transistor went from high to low which made the output of the flip-flop go from low to high. The low-to-high transition served as an interrupt to inform the computer that a touch had occurred. The computer then used the "checking status" line to find out which sensor issued the interrupt. The "checking status" line was connected to the $\bar{Q}$ output of the D flip-flop; therefore, a low signal indicated the source of the interrupt. Once the source of the interrupt was found, the corresponding flip-flop was cleared in order to issue the next interrupt.

B. Walking Up and Down the Slope

The robot was controlled by a set of programs which implemented the walking gaits and ran on the control computer. The biped initially stood on a level ground with the inclinations of the front foot and back foot being zero. It started walking using the transitional walking gait. At the end of the foot-landing phase when both the heel and the toe of the landing foot touched the ground, the robot checked the included angle α_i. If α_i was zero, the robot was still on the level ground; the transitional walking gait continued. Eventually, the robot came to an area where the included angle became nonzero. The robot started to go through the transition area and the transitional walking gait continued. After a while, the robot found that it was completely on the slope. We then used the slope walking gait to enable the robot to walk on the inclined board.

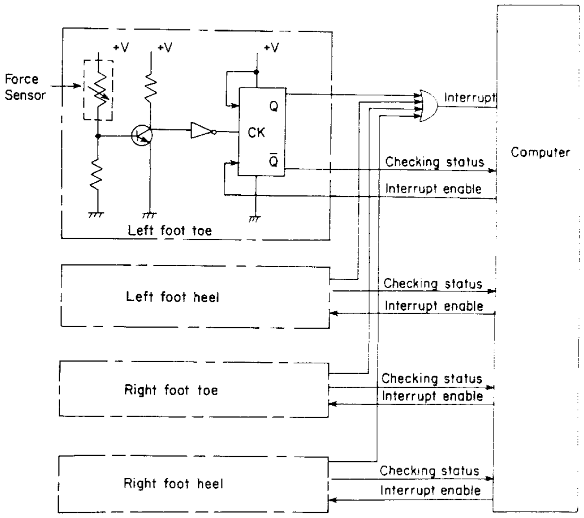


Fig. 12. Circuit for interfacing the force signal with the control computer.

Once the robot reached the top of the inclined board, we let the robot walk down the slope. The robot used the transitional walking gait to come down the slope with the initial inclination of the supporting foot being inherited from the previous slope walking. At the end of every step, the included angle was checked. After going through the area where the included angle was nonzero, the robot reached the level ground and the level walking gait started. For both walking up and down the slope, the robot had a speed of one step per second with the size of step being 5 in. By executing the two experiments of walking up and down the slope, all the three walking gaits, i.e., walking on the level ground, walking on the sloping surface, and transitional walking, were tested. It should be noted that we could use the transitional walking gait for the entire surface.

We have successfully conducted the above two experiments in our laboratory. The robot demonstrated a smooth and stable walking for both up and down slopes (Fig. 13). One may notice that in Fig. 13, the body is not in an upright position but instead is bent at an angle Δq (see Fig. 7). This body angle is determined by the steepness of the slope, i.e., the angle α (Fig. 7). In general, Δq should make the vertical projection of the center of gravity right in the middle of the support when the feet stand on the slope. The detailed relation between Δq and α, however, is determined by (4) and (5).

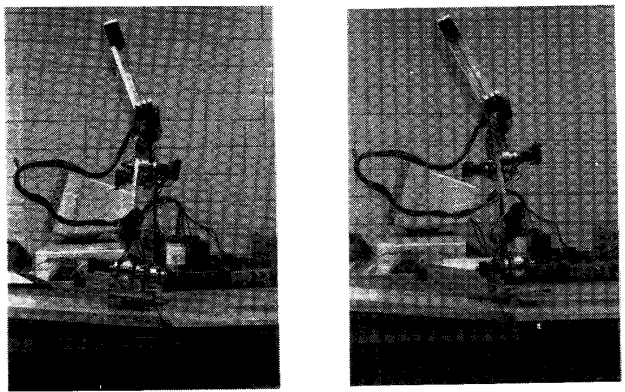

(a) (b)

Fig. 13. The SD-2 biped robot climbing and walking down the slope. (a) Climbing the slope. (b) Walking down the slope.

VI. Summary and Conclusions

A motion scheme for the SD-2 biped robot to climb sloping surfaces was proposed in this paper. The complete scheme consisted of three major aspects. The first aspect dealt with the detection and measurement of the inclination of the landing foot and the gradient of the slope. By using force sensors installed beneath the heel and toe, the biped robot could detect the sudden change of the ground and made compliant motions to adapt the landing foot to an unknown slope. Joint positions were then used in a newly developed algorithm to calculate the inclination of the landing foot as well as the gradient of the slope. The second aspect was about the motion scheme for slope walking. It turned out that slope walking only involved a simple modification to level walking, i.e., only the main-body motion needed to be reconsidered. The purpose of the modification was to compensate for the backward shift of the center of gravity while walking on a slope. The final aspect handled transitional walking. Four phases were included in the gait for transitional walking; namely, position adjusting, virtual slope walking, heel landing, and foot landing including phase A and phase B. The virtual slope walking phase employed the same gait as regular slope walking. The walking gaits for the other three phases, however, were newly developed. The experiment of the SD-2 biped robot walking up and down slopes were conducted in our laboratory. The results proved that the motion mechanism developed in this paper was valid.

Because of the complexity involved in biped locomotion, earlier studies were confined to only flat floor walking. This paper is the first to propose a slope walking mechanism for a biped robot. More significantly, the mechanism was experimentally verified. Although the proposed mechanism is based on static gait walking, the new concepts that we proposed for slope walking can be extended to quasi-dynamic and dynamic gaits as well. Consider the three aspects that we summarized in the previous paragraph, i.e., compliant motion for measuring the gradient of a slope, the motion scheme for slope walking, and transitional walking. We feel that they are as important to quasi-dynamic or dynamic gait as to static gait.

It is clear that when a robot climbs a slope, the gradient of the slope needs to be known in order to properly adjust the robot configuration regardless of the type of gait that the robot is using. The method that we proposed used a force sensor to make the feet comply with the ground. The same compliance mechanism is also valid for quasi-dynamic or dynamic walking since the robot still needs to adapt itself to the variation of the ground surface. When we use (1) and (2) to calculate the gradient of a slope, both feet need to stand on the ground. Quasi-dynamic and dynamic gaits also have a period of time when both feet contact the ground (if such a period does not exist, the gait is called running, not walking) although the period is shorter than in static walking. Therefore, our method for calculating the gradient of the slope is still valid for quasi-dynamic and dynamic walking provided the computation is very fast.

As far as slope walking is concerned, we proposed to use the modified gait of level walking. This philosophy should be valid for quasi-dynamic and dynamic walking as well. It is not realistic to develop a radically different gait for slope walking. For example, consider human locomotion. When we climb slopes, we lean forward but our body and legs keep the same basic walking pattern as in employed for level walking. It should be noted that in our robot, each leg only has two degrees of freedom. When both feet contact the ground (Fig. 7), the legs cannot shift the center of gravity; therefore, we have used the main body to adjust the center of gravity.

Finally, for transitional walking, our idea was to gradually adapt the main body to the variation of the surface using the compliance mechanism. In quasi-dynamic and dynamic walking, this transitional period is very short. Especially for dynamic walking, two feet never overlap, as shown in Fig. 4. If one foot already touches a slope, the other foot in the next step will be completely on the slope. As a result, transitional walking becomes even simpler in dynamic walking.

In conclusion, we feel that the significance of our work lies in the new mechanisms that are proposed for the first time in order to realize slope walking. The mechanism is basically for the SD-2 biped robot but can be extended to quasi-dynamic and dynamic walking robots. Nevertheless, many new problems still need to be studied if we want the robot to walk on uneven terrain. One immediate problem is walking from a level to a negative slope. The most difficult part will be involved in transitional walking because the feet may stand on the edge formed by the two levels. The edge, however, does not provide a solid support to the robot. This new problem will be addressed in future research.

REFERENCES

[1] A. A. Frank, "An approach to the dynamic analysis and synthesis of biped locomotion machines," *Med. Biol. Eng.*, vol. 8, pp. 465-476, 1970.

[2] M. Vukobratovic and A. A. Frank, "Legged locomotion studies," in *Advances in External Control of Human Extremities*, M. M. Gavrilovic, A. B. Wilson, Jr., Eds. Belgrade, Yugoslavia: Yugoslav Committee for Electronics and Automation, 1969, pp. 407-430.

[3] A. A. Frank, "On the stability of an algorithmic biped locomotion machine," *J. Terramech.*, vol. 8, no. 1, pp. 41-50, 1971.

[4] M. Vukobratovic, A. A. Frank, and D.. Juricic, "On the stability of biped locomotion," *IEEE Trans. Bio-Med. Eng.*, vol. BME-17, no. 1, pp. 25-36, Jan. 1970.

[5] A. A. Frank and M. Vukobratovic, "On the gait stability of biped machine," *IEEE Trans. Automat. Contr.*, vol. AC-15, no. 6, pp. 678-679, Dec. 1970.

[6] F. Gubina, H. Hemami, and R. B. McGhee, "On the dynamic stability of biped locomotion," *IEEE Trans. Bio-Med. Eng.*, vol. BME-21, no. 2, pp. 102-108, Mar. 1974.

[7] H. Hemami and B. F. Wyman, "Modeling and control of constrained dynamic systems with application to biped locomotion in the frontal plane," *IEEE Trans. Automat. Contr.*, vol. AC-24, no. 4, pp. 526-535, Aug. 1979.

[8] I. Kato, S. Ohteru, H. Kobayashi, K. Shirai, and A. Uchiyama, "Information-power machine with sense and limbs," in *Proc. 1st CISM-IFToMM Symp. on Theor. and Pract. of Robots and Manipula.* Berlin, Germany: Springer-Verlag, 1974.

[9] T. Kato, A. Takanishi, H. Ishikawa, and I. Kato, "The realization of quasi-dynamic walking by the biped machine," in *Proc. 4th CISM-IFToMM Symp. on Theor. and Pract. of Robots and Manipula.* Warsaw, Poland: PWN, 1983, pp. 341-351.

[10] A. Takanishi, M. Ishida, Y. Yamazaki, and I. Kato, "The realization of dynamic walking by the biped walking robot WL-10RD," in *Proc. 1985 Int. Conf. on Advanc. Robotics* (Tokyo, Japan, Sept. 9-10, 1985), pp. 459-466.

[11] H. Miura and H. Shimoyama, "Dynamical walk of biped locomotion," *Int. J. Robotics Res.*, vol. 3, no. 2, pp. 60-74, 1984.

[12] M. H. Raibert, *Legged Robots That Balance.* Cambridge, MA: MIT Press, 1986.

[13] S. Arimoto and F. Miyazaki, "Biped locomotion robots," in *JARECT*, vol. 12, *Computer Science & Technologies* T. Kitagawa, Ed. Amsterdam, The Netherlands: OHMSHA, LTD and North-Holland, 1984, pp. 194-205.

[14] Y. F. Zheng and F. R. Sias, Jr., "Design and motion control of practical biped robots," *Int. J. Robotics Automat.*, vol. 3, no. 2, pp. 70-77, 1988.

[15] R. B. McGhee, "Vehicular legged locomotion" in *Advances in Automation and Robotics*, G. N. Saridis, Ed. Greenwich, CT: Jai Press, 1983.

[16] "The force sensing resistor—A new tool in sensor technology," Tech. Rep. released by Interlink Electronics, Santa Barbara, CA 93105, June, 1986.

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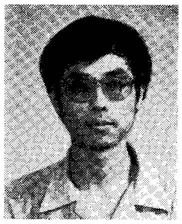

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