On The Stability of Anthropomorphic Systems *

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Communicated by Richard Bellman

ABSTRACT

This contribution treats definitions, dynamic aspects, and stability concepts of anthropomorphic systems. In addition to general conclusions about the new method of two-legged systems modelling, there are given some characteristic schemes of perturbed steady-gait regime stabilization.

METHOD OF ARTIFICIAL SYNERGY SYNTHESIS

The basic problem of the artificial locomotion-system synthesis consists in the elaboration of corresponding synergies, enabling one to reduce the number of control coordinates. This problem reduces to the elaboration of control algorithms, which have to ensure relative movement of the whole locomotion system or of its parts, according to some prescribed law.

It is known that the legged locomotion systems represent complex space systems with a great number of degrees of freedom. The attempt to synthetize a locomotion mechanism, reproducing with great similarity the human locomotion system, would lead to infinitely complex systems, particularly from the control standpoint.

It is sufficient to remind of the fact that the upper extremities of man contain 52 muscle pairs, the lower extremities 62 pairs, back—112 pairs, chest part—52 pairs, pelvic part—8 pairs. The neck contains 16 pairs and the head itself 25 pairs of muscles. The whole muscular system is able to control human motions with amazing complexity, enabling man to perform an almost arbitrary skeletal activity.

It is understandable that at the present level of technical progress it is

* The research reported in this paper was supported by the Mathematical Institute Beograd, Yugoslavia.

not possible to control an artificial system containing about 400 double-acting actuators (800 muscles).

Evidently, there arises the problem of how to reduce the total number of degrees of freedom at the dynamic level of the locomotion–manipulation system. In connection with this, there exist different attempts to reduce the dimensionality during the synthesis of the system for artificial skeletal activity, as compared with the natural system.

One of these [1] reduces the skeletal activity to a very limited number of movements, using the electrical stimulation of the natural locomotion system. Another approach studies the legged-locomotion dynamics on a rigid body model with six degrees of freedom [2, 3], moving under the effect of alternate force impulses. These impulses arise as the result of alternate leg contact with the supporting surface. The limitation of this approach evidently lies in the fact that leg masses have not been taken into account, although, as it is known, they represent roughly half of the total system mass.

In the proposed method the synergy of some type of gait is being realized as well as the synthesis of the compensating system, which is necessary to maintain the prescribed synergy [4, 5]. The synergy supposes the synchronization of the system parts relative movement and it is equivalent to introducing supplementary connections (constraints) in the locomotion-system mechanism. Due to these connections the total number of degrees of freedom diminishes considerably, and with a prescribed algorithm the system does not possess " freedom " in the classical sense; it moves according to a preselected law.

The synergy in question is being realized in different ways for the lower extremities and the upper part of the body. For the lower extremities a periodic algorithm is prescribed, imitating human gait. The upper body algorithm can be acquired from the gait repeatability conditions [4].

With the synthesis of artificial synergy, an important role is played by the dynamic links. Therefore, we will nominate some differential relations to be satisfied during the gait. They can be in the form of some relations, based upon reactions on the support surfaces of the feet.

In Fig. 1 an example of force distribution across the foot is given. As the load has the same sign all over the surface, it can be reduced to the resultant force R, the point of attack of which will be in the boundaries of the foot. Let the point on the surface of the foot, where the resultant R passes, be denoted as the zero-moment point, or ZMP in short.

In the case of the double-support phase, ZMP can find itself outside the support surface of the feet (dashed zone in Fig. 2). In the boundaries of this zone ZMP can move according to various laws, which define the gait to a considerable extent. The basic idea in the synthesis of synergy lies in prescribing the ZMP movement laws in advance. For instance, in the single-support phase, ZMP is in the center of the support surface of the foot, while in the double support-phase, it translates itself gradually or stepwise into the other foot surface center. If we denote with λ the point ZMP, according to d'Alambert's principle the sum of the external and

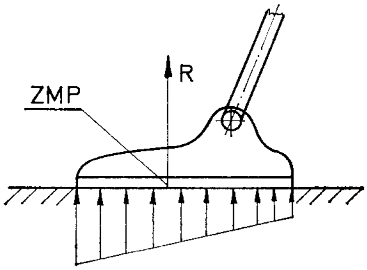

FIG. 1. Zero-moment point (ZMP).

inertial forces' moments relative to that point should be zero. Analogously, the law of the friction forces change can be prescribed, demanding for instance that the friction forces moment be zero at point, λ. This renders one more equation of dynamic connections.

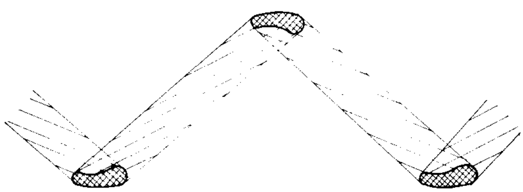

FIG. 2. Admissible region of ZMP position.

For the model considered we shall set motion laws of the model " legs " [that is, all coordinates $\beta_i(t)$, see Fig. 3] and from equations of dynamic connections with respect to the coordinates of the body upper portion (coordinates ψ, Γ). Then, differential equations of the dynamic connections (for more details see Eqs. 12 and 13) can be written in the following symbolic form:

$$Q\dot{Y} + Q_1 = 0$$
$$Y = (\psi, \vartheta, \dot{\psi}, \dot{\vartheta}) \tag{1}$$

where Y—vector of phase coordinates.

Matrices Q and Q_1 depend on vector Y and on set synergy $\beta_i(t)$, as well:

$$Q = Q(Y, \beta, \dot{\beta}, \ddot{\beta})$$
$$Q_1 = Q_1(Y, \beta, \dot{\beta}, \ddot{\beta}).$$

Let T be the step period. Let us denote

$$Y(0) = Y^o \qquad \text{and} \qquad Y(T) = Y^T$$

the phase coordinates at the beginning and end of the step.

Now the repeatability conditions can be presented by the following functional relation:

$$Y^T = \chi(Y^o). \tag{2}$$

Only those solutions of system 2, satisfying conditions 3 are of interest for consideration. The phase coordinate vector at the beginning of the step for that case we will denote with $\bar{Y}^o$.

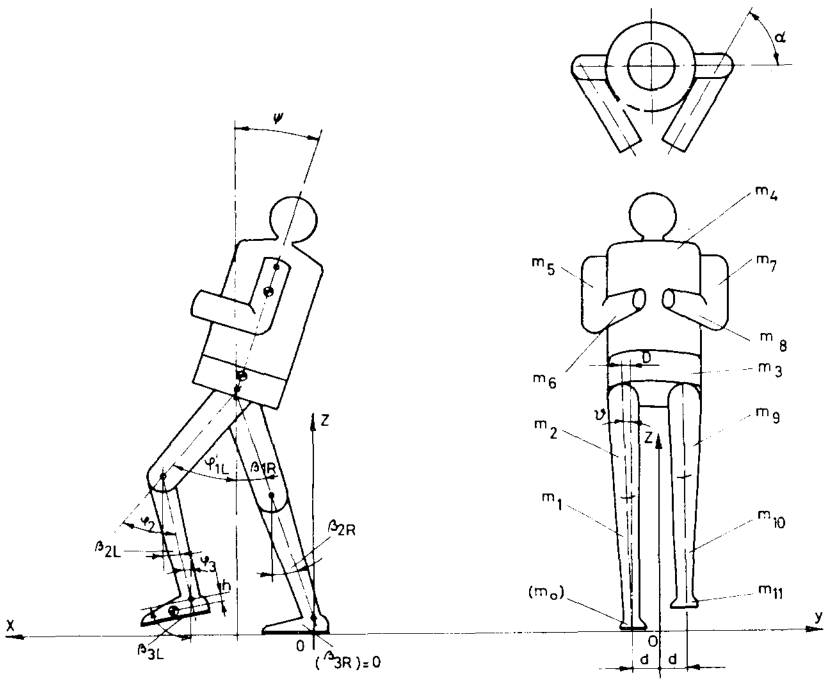

FIG. 3. Mechanical biped model.

Keeping in mind that the boundary conditions are given in the form of the functional relation 2 it is necessary to form an algorithm for automatic solution of the coupled system [1, 2], for the case when these solutions exist.

For this reason let us introduce the performance index for fulfilling conditions 2.

Let $\tilde{Y}(t)$ be some solution of 1 not satisfying relation 2. As before, let us denote

$$\tilde{Y}(0) = \tilde{Y}^o \qquad \tilde{Y}(T) = \tilde{Y}^T.$$

As the performance index, let us introduce the relation:

$$J = \|\tilde{Y}^T - \chi(\tilde{Y}^o)\|. \tag{3}$$

As $\tilde{Y}^T$ and $\tilde{Y}^o$ are correlated by differential Eq. 2, J is a function of $\tilde{Y}^o$ only:

$$J = J(\tilde{Y}^o).$$

It is evident that the repeatability conditions are now equivalent to:

$$J(\bar{Y}^o) = \min_{\tilde{Y}^o} J(\tilde{Y}^o) = 0. \tag{4}$$

In order to solve 4, the gradient method can be applied:

$$\tilde{Y}^0_{i+1} = \tilde{Y}^0_i - \varepsilon \nabla J \tag{5}$$

where $\nabla J = \operatorname{grad} J(\tilde{Y}^o)$

i — number of iteration steps.

In the cases when the phase coordinate vector $\tilde{Y}^o$ is sufficiently near to the nominal value $\bar{Y}^o$, the following local method can be introduced.

Let the deviation $\Delta Y^o = \bar{Y}^o - \tilde{Y}^o$ be sufficiently small. This deviation causes a small deviation $\Delta Y^T = \bar{Y}^T - \tilde{Y}^T$ at the end of the step. The expression 2 can be written as:

$$\bar{Y}^T + \Delta Y^T = \chi(\bar{Y}^o + \Delta Y^o). \tag{6}$$

The correlation between the deviation at the beginning and end of the step can be expressed as:

$$\Delta Y^T = \frac{\partial Y^T}{\partial Y^0} \Delta Y^0 \tag{7}$$

where the members of the matrix $|\partial Y^T / \partial Y^o|$ are calculated in the point $\bar{Y}^o = \tilde{Y}^o$.

By solving systems 6 and 7 the sought value $\Delta \bar{Y}^o$ can be found

$$\Delta \bar{Y}^o = \phi(\tilde{Y}^o). \tag{8}$$

If J is changing strictly monotonously, the method explained can be used also in the cases in which the value of the phase coordinate $\tilde{Y}^o$ differs considerably from the nominal value $\bar{Y}^o$. Obtaining the repeatability conditions in such a case is effected more efficiently by the gradient method (see Eq. 5). The monotonous change of J can be ascertained by choosing ε sufficiently small in the following relation:

$$\tilde{Y}^0_{i+1} = \tilde{Y}^0_i + \varepsilon \phi(\tilde{Y}^0_i). \tag{9}$$

In order to accelerate the process of obtaining the repeatability conditions, it is advisable to combine criteria 6 and 10. The transfer from criterion 5 to 9 should be done when J becomes smaller than J^*, where J^* is a predetermined value of the performance index 3.

In compliance with the physical nature of gait, condition 2 can be written in the form:

$$Y^o = \eta Y^T \tag{10}$$

where the lower index denotes the number of the phase coordinate. In the general case matrix η has the form:

$$\eta = \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & \cdot & & \mathrm{O} \\ & & & \cdot & \\ & \mathrm{O} & & & \cdot \\ & & & & 1 \end{bmatrix}. \tag{11}$$

SYNERGY GENERATION

In order to investigate gait stability we are going to form the mathematical model describing the locomotion structure dynamics represented in Fig. 3.

The upper part of the locomotion structure is regarded as being in the form of an inverted pendulum. The lower extremities have feet and each extremity has three degrees of freedom; the segments are interconnected by simple joints. For leg movement a " real " gait algorithm is adopted. In Fig. 4 some of the diagrams representing gait upon level ground, upstairs and downstairs, which have been synthesized from data acquired from biometrical investigations, are given. The chosen gait types are characterized by a very " smooth " behavior of the locomotion-system

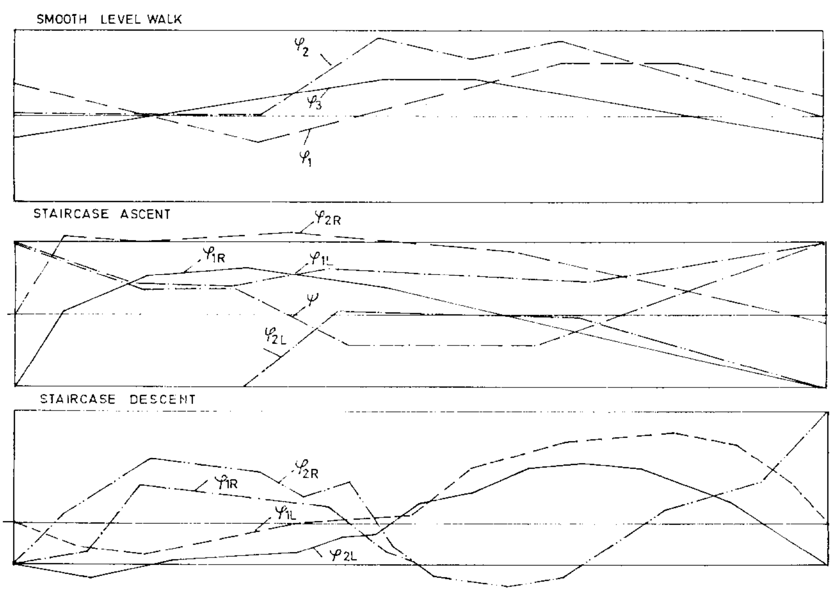

FIG. 4. Typical synthetic gait algorithms.

pelvic part. This supposition is of a purely practical nature, because the applicability of these results to exoskeleton-type biped robots is kept in mind.

According to the chosen gait algorithm, the supporting foot transfers from heel to toes as illustrated in Fig. 5. In this case, three phases can be separated, corresponding to the positions in Fig. 5. Let us designate with t_{ab} the moment of support passing from heel to the whole foot and with t_{bc} the corresponding moment of support passing from whole foot to the toes $(0 < t_{ab} < t_{bc} < T/2)$ where T—full step period.

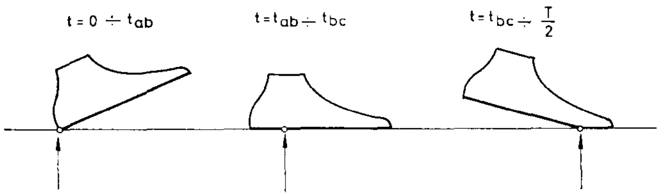

FIG. 5. Supporting point changes.

During the half-period, the zero-moment point "jumps" three times to a new position: at the end of the first phase from the heel to the "center" of the foot, and at the end of the second phase from that position to the toes (Fig. 5). At the end of the half-period, the zero-moment point is shifted under the other foot, which is in contact with the ground. It should be stressed that such a transfer of the point of support has made the gait smoother to a certain extent. However, an even more natural gait* can be realized by prescribing the zero-moment point trajectory corresponding to the double-support phase; this approach is not treated here.

Under the supposition that we dispose with the kinematic algorithm (chosen-gait type) and the zero-moment point trajectory (ZMP) we can proceed to obtain the upper part dynamic algorithm. Let us write the equations of dynamic connections using d'Alambert's principle. These equations are formed according to the general form of Eq. 1. For the chosen gait algorithm (Fig. 4), angular displacements of the structure pelvic part are practically nonexistent. If we additionally suppose that the friction moment on the supporting foot is sufficiently great to ensure planar motion of the lower extremities, we can neglect the third differential equation of system 1, describing the system dynamic equilibrium round the z-axis. Here x_i, y_i, z_i are coordinates of the center of mass of the i-th segment. Other denotations are evident from Fig. 3.

* In this case, the gait comprises the movement of the lower extremities themselves (kinematic algorithm), as well as the movement of the locomotion system compensation part (dynamic algorithm).

$$M_y \equiv \ddot{\vartheta}\left[\sum_{i=1}^{11} m_i(V_i z_i - R_i x_i)\right] + \ddot{\bar{\psi}}\left[\sum_{i=1}^{11} m_i(W_i z_i - S_i x_i)\right.$$
$$\left. + J_{y4} + J_{y5} + J_{y6} + J_{y7} + J_{y8}\right]$$
$$+ \sum_{i=1}^{11} m_i(P_i z_i - T_i x_i) - g\sum_{i=1}^{11} m_i x_i$$
$$+ J_{y1}\ddot{\beta}_{2L} + J_{y2}\ddot{\beta}_{1L} + J_{y9}\ddot{\beta}_{1R} + J_{y10}\ddot{\beta}_{2R} + J_{y11}\ddot{\beta}_{3R} = 0. \tag{12}$$

$$M_x \equiv \ddot{\vartheta}\left[\sum_{i=1}^{11} m_i(R_i y_i - A_i z_i) + J_{x1} + J_{x2} + J_{x9} + J_{x10} + J_{x11}\right]$$
$$+ \ddot{\bar{\psi}}\sum_{i=1}^{11} m_i S_i y_i + \sum_{i=1}^{11} m_i(T_i y_i - C_i z_i) + g\sum_{i=1}^{11} m_i y_i = 0, \tag{13}$$

where

$$
\begin{aligned}
V_1 &= -a\sin\vartheta\sin\beta_{2L},\\
V_2 &= 2V_1 - b\sin\vartheta\sin\beta_{1L},\\
V_3 &= V_2 - b\sin\vartheta\sin\beta_{1L},\\
V_4 &= V_3, \quad V_5 = V_3, \quad V_6 = V_3, \quad V_7 = V_5, \quad V_8 = V_6\\
V_9 &= V_3 - b\sin\vartheta\sin\beta_{1R},\\
V_{10} &= V_3 - (2b\sin\beta_{1R} + a\sin\beta_{2R})\sin\vartheta,\\
V_{11} &= V_3 - (2b\sin\beta_{1R} + 2a\sin\beta_{2R} + h\sin\beta_{3R})\sin\vartheta,\\
W_1 &= 0, \quad W_2 = 0, \quad W_3 = 0,\\
W_4 &= c\cos\bar{\psi}, \quad W_5 = (R-e)\cos\bar{\psi},\\
W_6 &= (R-2e)\cos\bar{\psi} - S\cos\alpha\sin\bar{\psi},\\
W_7 &= W_5, \quad W_8 = W_6, \quad W_9 = 0, \quad W_{10} = 0, \quad W_{11} = 0,\\
P_1 &= -a\dot{\vartheta}^2\cos\vartheta\sin\beta_{2L} - a\dot{\vartheta}\dot{\beta}_{2L}\sin\vartheta\cos\beta_{2L}\\
&\quad + a\ddot{\beta}_{2L}\cos\vartheta\cos\beta_{2L} - a\dot{\beta}_{2L}\dot{\vartheta}\sin\vartheta\cos\beta_{2L}\\
&\quad - a\dot{\beta}_{2L}^2\cos\vartheta\sin\beta_{2L},\\
P_2 &= 2P_1 - b\dot{\vartheta}^2\cos\vartheta\sin\beta_{1L} - b\dot{\vartheta}\dot{\beta}_{1L}\sin\vartheta\cos\beta_{1L}\\
&\quad + b\ddot{\beta}_{1L}\cos\vartheta\cos\beta_{1L} - b\dot{\beta}_{1L}\dot{\vartheta}\sin\vartheta\cos\beta_{1L} - b\dot{\beta}_{1L}^2\cos\vartheta\sin\beta_{1L},\\
P_3 &= P_2 - \dot{\vartheta}^2 b\cos\vartheta\sin\beta_{1L} - \dot{\vartheta}\dot{\beta}_{1L} b\sin\vartheta\cos\beta_{1L} + b\ddot{\beta}_{1L}\cos\vartheta\cos\beta_{1L}\\
&\quad - b\dot{\beta}_{1L}\dot{\vartheta}\sin\vartheta\cos\beta_{1L} - b\dot{\beta}_{1L}^2\cos\vartheta\sin\beta\\
P_4 &= P_3 - c\dot{\bar{\psi}}^2\sin\bar{\psi}, \quad P_5 = P_3 - (R-e)\dot{\bar{\psi}}^2\sin\bar{\psi},\\
P_6 &= P_3 - (R-2e)\dot{\bar{\psi}}^2\sin\bar{\psi} - \dot{\bar{\psi}}^2\cos\alpha\cos\bar{\psi},\\
P_7 &= P_5, \quad P_8 = P_6,\\
P_9 &= P_3 - b\dot{\vartheta}^2\cos\vartheta\sin\beta_{1R} - 2b\dot{\vartheta}\dot{\beta}_{1R}\sin\vartheta\cos\beta_{1R}\\
&\quad + b\ddot{\beta}_{1R}\cos\vartheta\cos\beta_{1R} - b\dot{\beta}_{1R}^2\cos\vartheta\sin\beta_{1R},\\
P_{10} &= P_3 + (2b\ddot{\beta}_{1R}\cos\beta_{1R} - 2b\dot{\beta}_{1R}^2\sin\beta_{1R} + a\ddot{\beta}_{2R}\cos\beta_{2R}\\
&\quad - a\dot{\beta}_{2R}^2\sin\beta_{2R})\cos\vartheta - 2\dot{\vartheta}(2b\dot{\beta}_{1R}\cos\beta_{1R}\\
&\quad + a\dot{\beta}_{2R}\cos\beta_{2R})\sin\vartheta - \dot{\vartheta}^2(2b\sin\beta_{1R} + a\sin\beta_{2R})\cos\vartheta,
\end{aligned}
$$

$$
\begin{aligned}
P_{11} &= P_3 + (2b\ddot{\beta}_{1R}\cos\beta_{1R} - 2b\dot{\beta}_{1R}^2\sin\beta_{1R} + 2a\ddot{\beta}_{2R}\cos\beta_{2R} \\
&\quad - 2a\dot{\beta}_{2R}^2\sin\beta_{2R} + h\ddot{\beta}_{3R}\cos\beta_{3R} - h\dot{\beta}_{3R}^2\sin\beta_{3R})\cos\vartheta \\
&\quad - 2\dot{\vartheta}(2b\,\dot{\beta}_{1R}\cos\beta_{1R} + 2a\dot{\beta}_{2R}\cos\beta_{2R} + h\dot{\beta}_{3R}\cos\beta_{3R})\sin V \\
&\quad - \dot{\vartheta}^2(2b\sin\beta_{1R} + 2a\sin\beta_{2R} + h\sin\beta_{3R})\cos\vartheta, \\
A_1 &= a\cos\beta_{2R}\cos\vartheta, \\
A_2 &= 2A_1 + b\cos\beta_{1L}\cos\vartheta, \\
A_3 &= A_2 + b\cos\beta_{1L}\cos\vartheta, \\
A_4 &= A_3, \quad A_5 = A_3, \quad A_6 = A_3, \quad A_7 = A_3, \quad A_8 = A_3, \\
A_9 &= A_3 - b\cos\beta_{1R}\cos\vartheta, \\
A_{10} &= A_3 - (2b\cos\beta_{1R} + a\cos\beta_{2R})\cos\vartheta, \\
A_{11} &= A_3 - (2a\cos\beta_{2R} + 2b\cos\beta_{1R} + h\cos\beta_{3R})\cos\vartheta, \\
C_1 &= -\,a\ddot{\beta}_{2L}\sin\beta_{2L}\sin\vartheta - a\dot{\beta}_{2L}^2\cos\beta_{2L}\sin\vartheta - a\dot{\beta}_{2L}\dot{\vartheta}\sin\beta_{2L}\cos\vartheta \\
&\quad - a\dot{\vartheta}\dot{\beta}_{2L}\sin\beta_{2L}\cos\vartheta - a\dot{\vartheta}^2\cos\beta_{2L}\sin\vartheta, \\
C_2 &= 2C_1 - b\ddot{\beta}_{1L}\sin\beta_{1L}\sin\vartheta - b\dot{\beta}_{1L}^2\cos\beta_{1L}\sin\vartheta \\
&\quad - 2b\dot{\beta}_{1L}\dot{\vartheta}\sin\beta_{1L}\cos\vartheta - b\dot{\vartheta}^2\cos\beta_{1L}\sin\vartheta, \\
C_3 &= C_2 - b\ddot{\beta}_{1L}\sin\beta_{1L}\sin\vartheta - b\dot{\beta}_{1L}^2\cos\beta_{1L}\sin\vartheta \\
&\quad - 2b\dot{\beta}_{1L}\dot{\vartheta}\sin\beta_{1L}\cos\vartheta - b\dot{\vartheta}^2\cos\beta_{1L}\sin\vartheta, \\
C_4 &= C_3, \quad C_5 = C_3, \quad C_6 = C_3, \quad C_7 = C_3, \quad C_8 = C_3, \\
C_9 &= C_3 + b\ddot{\beta}_{1R}\sin\beta_{1R}\sin\vartheta + b\dot{\beta}_{1R}^2\cos\beta_{1R}\sin\vartheta \\
&\quad + 2b\dot{\beta}_{1R}\dot{\vartheta}\sin\beta_{1R}\cos\vartheta + b\dot{\vartheta}^2\cos\beta_{1R}\sin\vartheta, \\
C_{10} &= C_3 + (2b\ddot{\beta}_{1R}\sin\beta_{1R} + 2b\dot{\beta}_{1R}^2\cos\beta_{1R} + a\ddot{\beta}_{2R}\sin\beta_{2R} \\
&\quad + a\dot{\beta}_{2R}^2\cos\beta_{2R})\sin\vartheta + 2\dot{\vartheta}(2b\dot{\beta}_{1R}\sin\beta_{1R} + a\dot{\beta}_{2R}\sin\beta_{2R})\cos\vartheta \\
&\quad + \dot{\vartheta}^2(2b\cos\beta_{1R} + a\cos\beta_{2R})\sin\vartheta, \\
C_{11} &= C_3 + (2a\ddot{\beta}_{2R}\sin\beta_{2R} + 2a\dot{\beta}_{2R}^2\cos\beta_{2R} + 2b\ddot{\beta}_{1R}\sin\beta_{1R} \\
&\quad + 2b\dot{\beta}_{1R}^2\cos\beta_{1R} + h\ddot{\beta}_{3R}\sin\beta_{3R} + h\dot{\beta}_{3R}^2\cos\beta_{3R})\sin\vartheta \\
&\quad + 2\dot{\vartheta}(2a\dot{\beta}_{2R}\sin\beta_{2R} + 2b\dot{\beta}_{1R}\sin\beta_{1R} + h\dot{\beta}_{3R}\sin\beta_{3R})\cos\vartheta \\
&\quad + \dot{\vartheta}^2(2a\cos\beta_{2R} + 2b\cos\beta_{1R} + h\cos\beta_{3R})\sin\vartheta, \\
R_1 &= -\,a\cos\beta_{2L}\sin\vartheta, \\
R_2 &= -(2a\cos\beta_{2L} + b\cos\beta_{1L})\sin\vartheta, \\
R_3 &= R_2 - b\cos\beta_{1L}\sin\vartheta, \\
R_4 &= R_3, \quad R_5 = R_3, \quad R_6 = R_3, \quad R_7 = R_5, \quad R_8 = R_6, \\
R_9 &= R_3 + b\cos\beta_{1R}\sin\vartheta, \quad R_{10} = R_9 + (b\cos\beta_{1R} + a\cos\beta_{2R})\sin\vartheta \\
R_{11} &= R_{10} + a\cos\beta_{2R}\sin\vartheta, \\
S_1 &= S_2 = S_3 = 0, \quad S_4 = -\,c\sin\bar{\psi}, \\
S_5 &= -(R-e)\sin\bar{\psi}, \quad S_6 = -\,[(R-2e)\sin\bar{\psi} + s\cos\alpha\cos\bar{\psi}], \\
S_7 &= S_5, \quad S_8 = S_6, \quad S_9 = S_{10} = S_{11} = 0, \\
T_1 &= -\,a[\ddot{\beta}_{2L}\sin\beta_{2L}\cos\vartheta + \dot{\beta}_{2L}^2\cos\beta_{2L}\cos\vartheta - 2\dot{\beta}_{2L}\dot{\vartheta}\sin\beta_{2L}\sin\vartheta \\
&\quad + \dot{\vartheta}^2\cos\beta_{2L}\cos\vartheta], \\
T_2 &= -\,(2a\ddot{\beta}_{2L}\sin\beta_{2L} + 2a\dot{\beta}_{2L}^2\cos\beta_{2L} + b\ddot{\beta}_{1L}\sin\beta_{1L} \\
&\quad + b\dot{\beta}_{1L}^2\cos\beta_{1L})\cos\vartheta + 2\dot{\vartheta}(2a\dot{\beta}_{2L}\sin\beta_{2L} \\
&\quad + b\dot{\beta}_{1L}\sin\beta_{1L})\sin\vartheta - \dot{\vartheta}^2(2a\cos\beta_{2L} + b\cos\beta_{1L})\cos\vartheta,
\end{aligned}
$$

$$
\begin{aligned}
T_3 &= T_2 - b\ddot{\beta}_{1L}\sin\beta_{1L}\cos\vartheta - b\dot{\beta}_{1L}^2\cos\beta_{1L}\cos\vartheta \\
&\quad + 2b\dot{\vartheta}\dot{\beta}_{1L}\sin\beta_{1L}\sin\vartheta - b\dot{\vartheta}^2\cos\beta_{1L}\cos\vartheta, \\
T_4 &= T_3 - c\dot{\bar{\psi}}^2\cos\bar{\psi}, \\
T_5 &= T_3 - \dot{\bar{\psi}}^2(R - e)\cos\bar{\psi}, \\
T_6 &= T_3 - \dot{\bar{\psi}}^2[(R - 2e)\cos\bar{\psi} - S\cos\alpha\sin\bar{\psi}], \\
T_7 &= T_5, \quad T_8 = T_6, \\
T_9 &= T_3 + b\ddot{\beta}_{1R}\sin\beta_{1R}\cos\vartheta + b\dot{\beta}_{1R}^2\cos\beta_{1R}\cos\vartheta \\
&\quad - 2b\dot{\vartheta}\dot{\beta}_{1R}\sin\beta_{1R}\sin\vartheta + b\dot{\vartheta}^2\cos\beta_{1R}\cos\vartheta, \\
T_{10} &= T_9 - 2\dot{\vartheta}(b\dot{\beta}_{1R}\sin\beta_{1R} + a\dot{\beta}_{2R}\sin\beta_{2R})\sin\vartheta \\
&\quad + (b\ddot{\beta}_{1R}\sin\beta_{1R} + b\dot{\beta}_{1R}^2\cos\beta_{1R} - a\ddot{\beta}_{2R}\sin\beta_{2R} \\
&\quad - a\dot{\beta}_{2R}^2\cos\beta_{2R})\cos\vartheta + \dot{\vartheta}^2(b\cos\beta_{1R} + a\cos\beta_{2R})\cos\vartheta, \\
T_{11} &= T_{10} + a\ddot{\beta}_{2R}\sin\beta_{2R}\cos\vartheta + a\dot{\beta}_{2R}^2\cos\beta_{2R}\cos\vartheta \\
&\quad - 2a\dot{\vartheta}\dot{\beta}_{2R}\sin\beta_{2R}\sin\vartheta + a\dot{\vartheta}^2\cos\beta_{2R}\cos\vartheta.
\end{aligned}
$$

These equations have been written for a support point when ZMP corresponds to the contact with the "whole" foot. As ZMP displaces itself according to the already mentioned law (Fig. 5), the translation of the coordinate system should be taken care of.

It has to be noted, as well, that the eqs. 12 and 13 for the model shown in Fig. 3, are presented for the purpose of illustrating the method of set synergy. At the same time, such a model can also completely satisfy the practical objectives of locomotion study.

To obtain a complete mathematical model it is necessary in compliance with Fig. 5 to change the z coordinate of the center of gravity, that is, foot joint, while the coordinates x_i due to the change in point of moment should be translated by $+l_1$ (contact of heel) and $-l_2$ (contact by toes) with respect to the adopted zero moment point that corresponds to the contact by full foot. Finally, when the support passes to the other foot,

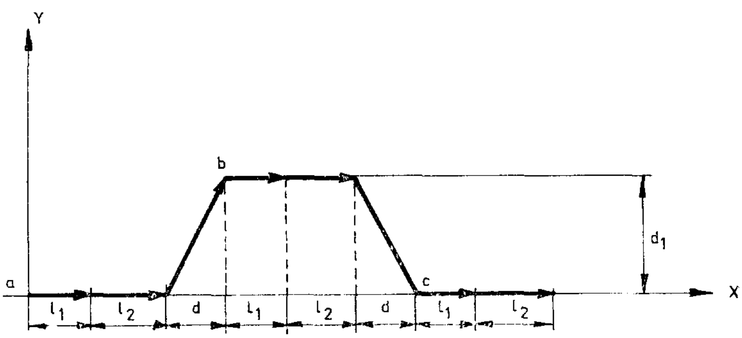

FIG. 6. Schematic presentation of coordinates changes.

the x-coordinates should be reduced by the value d and the y-coordinates should change their value abruptly by d_1 (Fig. 6). The segment abc in Fig. 6 corresponds to a full step (period T), whilst segment ab corresponds to a half step.

Due to system symmetry only half of the step can be considered. The repeatability conditions in that case will be:

$$Y^o = \begin{bmatrix} -1 & & & 0 \\ & 1 & & \\ 0 & & -1 & \\ & & & 1 \end{bmatrix} Y^T, \tag{14}$$

where

$$Y = \begin{Bmatrix} \vartheta \\ \psi \\ \dot{\vartheta} \\ \dot{\psi} \end{Bmatrix}.$$

In this case the performance index J and the expressions for J have the form:

$$\begin{aligned} J(Y^0) &= [(Y_1^0 + Y_1^T)^2 + (Y_2^0 - Y_2^T)^2 + (Y_3^0 + Y_3^T)^2 + (Y_4^0 - Y_4^T)^2]^{1/2} \\ \nabla J &= \{\nabla_1 J, \nabla_2 J, \nabla_3 J, \nabla_4 J\}, \end{aligned} \tag{15}$$

$$\begin{aligned} \nabla_1 J &= \left[(Y_1^o + Y_1^T)\left(1 + \frac{\Delta Y_1^T}{\Delta Y_1^o}\right) - (Y_2^o - Y_2^T)\frac{\Delta Y_2^T}{\Delta Y_1^o} + \right. \\ &\qquad \left. + (Y_3^o + Y_3^T)\frac{\Delta Y_3^T}{\Delta Y_1^o} - (Y_4^o - Y_4^T)\frac{\Delta Y_4^T}{\Delta Y_1} \right] \Big/ J, \\ \nabla_2 J &= \left[(Y_1^o + Y_1^T)\frac{\Delta Y_1^T}{\Delta Y_2^o} + (Y_2^o - Y_2^T)\left(1 - \frac{\Delta Y_2^T}{\Delta Y_2^o}\right) + \right. \\ &\qquad \left. + (Y_3^o + Y_3^T)\frac{\Delta Y_3^T}{\Delta Y_2^o} - (Y_4^o - Y_4^T)\frac{\Delta Y_4^T}{\Delta Y_2^o} \right] \Big/ J, \\ \nabla_3 J &= \left[(Y_1^o + Y_1^T)\frac{\Delta Y_1^T}{\Delta Y_3^o} - (Y_2^o - Y_2^T)\frac{\Delta Y_2^T}{\Delta Y_3^o} \right. \\ &\qquad \left. + (Y_3^o + Y_3^T)\left(1 + \frac{\Delta Y_3^T}{\Delta Y_3^o}\right) - (Y_4^o - Y_4^T)\frac{\Delta Y_4^T}{\Delta Y_3^o} \right] \Big/ J, \\ \nabla_4 J &= \left[(Y_1^o + Y_1^T)\frac{\Delta Y_1^T}{\Delta Y_4^o} - (Y_2^o - Y_2^T)\frac{\Delta Y_2^T}{\Delta Y_4^o} \right. \\ &\qquad \left. + (Y_3^o + Y_3^T)\frac{\Delta Y_3^T}{\Delta Y_4^o} + (Y_4^o - Y_4^T)\left(1 - \frac{\Delta Y_4^T}{\Delta Y_4^o}\right) \right] \Big/ J. \end{aligned} \tag{16}$$

Starting from these expressions, the function ϕ from the relation 8 becomes:

$$\phi = [A]^{-1}q$$

where

$$q = [-Y_1^T - Y_1^o,\ Y_2^o - Y_2^T,\ -Y_3^T - Y_3^o,\ Y_4^o - Y_4^T]', \tag{17}$$

and

$$A = \begin{bmatrix} \frac{\Delta Y_1^T}{\Delta Y_1^o}+1 & \frac{\Delta Y_1^T}{\Delta Y_2^o} & \frac{\Delta Y_1^T}{\Delta Y_3^o} & \frac{\Delta Y_1^T}{\Delta Y_4^o} \\ \frac{\Delta Y_2^T}{\Delta Y_1^o} & \frac{\Delta Y_2^T}{\Delta Y_2^o}-1 & \frac{\Delta Y_2^T}{\Delta Y_3^o} & \frac{\Delta Y_2^T}{\Delta Y_4^o} \\ \frac{\Delta Y_3^T}{\Delta Y_1^o} & \frac{\Delta Y_3^T}{\Delta Y_2^o} & \frac{\Delta Y_3^T}{\Delta Y_3^o}+1 & \frac{\Delta Y_3^T}{\Delta Y_4^o} \\ \frac{\Delta Y_4^T}{\Delta Y_1^o} & \frac{\Delta Y_4^T}{\Delta Y_2^o} & \frac{\Delta Y_4^T}{\Delta Y_3^o} & \frac{\Delta Y_4^T}{\Delta Y_4^o}-1 \end{bmatrix}. \tag{18}$$

By simultaneously solving systems 12–14 and the sensitivity Eqs. 8, using expressions 17 and 18, the locomotion system upper body algorithm can be obtained, satisfying the repeatability conditions.

On the basis of the described method, repeatability conditions can be obtained, representing in fact the calculated synergy of the rest of the system (dynamic algorithm), based upon the prescribed synergy of one part of the system (kinematic algorithm). One of the characteristic diagrams in the phase plane of two compensating coordinates ψ and ϑ in the form of a closed curve, represents in fact the satisfied repeatability conditions (Fig. 7). The curve has been obtained for characteristic parameters of the locomotion system $S = 1$, $T = 2$ sec, where S is the coefficient of kinematic algorithm amplitude scaling (parameter of step length), and T is the step period (parameter of gait speed).

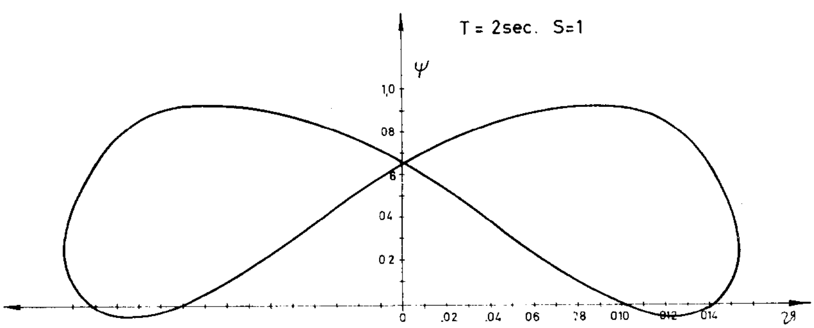

FIG. 7. Nominal gait trajectory for biped model with fixed upper extremities.

In the preceding text it was shown in short how the synergy of the complete system is being formed. For one part the synergy was prescribed and for the other part it was calculated using the dynamic analysis. Consequently we possess the relative coordinates $\varphi_i(t)$, i.e. the complete synergy ensuring periodic gait. This synergy has been defined for "ideal" conditions, under the supposition that no perturbations are acting on the locomotion system under consideration.

Under ideal conditions there exist periodic change laws $\beta_i(t)$, corresponding to the $\varphi_i(t)$ laws, where $\beta_i(t)$ as compared with $\varphi_i(t)$ define the positions of the locomotion-system elements in relation to a fixed absolute coordinate system. For this reason, let us introduce the concept of internal synergy for $\varphi_i(t)$ and external synergy for $\beta_i(t)$.

In the event of perturbation, even with very strict fulfilling of the internal synergy $\varphi_i(t)$, the external synergy can be perturbed. For instance, the whole system can rotate around the supporting foot, which causes the angles $\beta_i(t)$ to change.

For illustration purposes, the side view of the locomotion system is shown in Fig. 8. Due to some external perturbation the model can pass to some position, in which support is on the edge of the foot. Let us denote the angle between the foot and support with ξ. If in the case of absence of perturbations the external synergy $\beta_i(t)$ was defined by the internal synergy $\varphi_i(t)$ only, for instance for the model upper part:

$$\beta = \frac{\pi}{2} - \varphi,$$

in the presence of perturbations, $\beta_i(t)$ becomes

$$\beta = \frac{\pi}{2} - \varphi - \xi.$$

If due to any reason the internal synergy $\varphi_i(t)$ is not being realized, this state reflects itself on the external synergy $\beta_i(t)$. On the other hand, external synergy (and not internal) defines a repeatable gait in relation to an absolute coordinate system.

Consequently, under stable gait we will understand such a gait, in which external synergy tends to the "ideal" synergy, which has been defined in the absence of perturbations.

Let us now formalize this concept and make it more precise. We introduce the following designations. With the upper index "o" denote the coordinate change laws, obtained from ideal conditions. We will call them "ideal" coordinates. Consequently, φ^o and β^o represent the ideal synergy, whilst φ and β correspond to the real synergy.

Let us suppose that for some reason the internal synergy of the system has been perturbed and that φ differs from φ^o. Two cases can be distinguished. In the first one, the model can possess a stability margin [8] due to its geometrical properties, i.e. it will be tending to the ideal external synergy in the case of small perturbations.

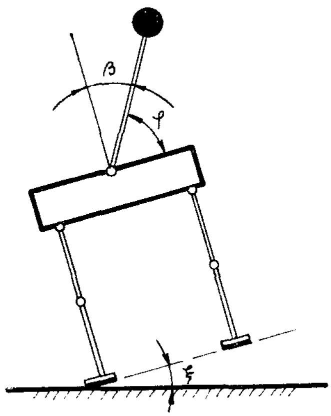

FIG. 8. Side view of the locomotion system.

The second case is characterized by the fact, that the stability reserve is insufficient (or even nonexistent) so for maintaining dynamic equilibrium special compensating movements of the system are needed. The systems with stability margin will be treated later. Now we will examine the second case.

GAIT STABILITY AND CONTROL ALGORITHMS

The compensating actions of the system represent internal forces and modify the $\varphi(t)$. In the other words, an influence on the external synergy is possible only by means of an internal synergy change, representing a specificity of the legged systems under consideration.

Here we can distinguish two basically different cases. In the first one, we can choose a new internal synergy φ^o, starting from the real external synergy β, in such a way, that the real synergy corresponds to the new ideal synergy β^o.

In the second case, we can change the internal synergy in such a way, that the external synergy approaches the ideal synergy β^o.

Each of these two cases will be more thoroughly examined. Be any of the two compensation methods adopted, there remains the criterion problem of deviation of the real external synergy from the ideal synergy.

The task of the elaboration of such a criterion presents one of the basic problems in gait-stability analysis.

The actuators produce forces (driving torques) during compensation, and as a direct consequence, change in the accelerations of the locomotion system parts is produced. To judge the efficiency of the compensating actions, the criterion must contain, besides the coordinates, their first and second derivatives too.

To achieve this, let us introduce the following criterion of the deviation between the real and ideal synergy:

$$J_i(t) = c_{oi}[\beta_i(t) - \beta_i^o(t)] + c_{1i}[\dot{\beta}_i(t) - \dot{\beta}_i^o] + c_{2i}[\ddot{\beta}_i(t) - \ddot{\beta}_i^o(t)], \tag{19}$$

where c_{oi}, c_{1i}, c_{2i} are weighting coefficients, and i's segment number of the anthropomorphic model.

The performance index for each separate element of the locomotion system will be considered as a component of vector J:

$$J = (J_1, \ldots, J_n)'. \tag{20}$$

The task of the compensating system is to reduce the value of the performance index to minimum.

Now it is necessary to find such compensating moments in the driving system, able to reduce to zero the performance index 19 in the course of a sufficiently small time interval τ.

Let us write the relations between β and $\dot{\beta}$ in the time moments t_1 and $t_2 = t_1 + \tau$:

$$\begin{aligned} \beta(t_2) &= \beta(t_1) + \dot{\beta}(t_1) \,.\, \tau, \\ \dot{\beta}(t_2) &= \dot{\beta}(t_1) + \ddot{\beta}(t_1) \,.\, \tau. \end{aligned} \tag{21}$$

Having in mind the size of the time interval τ we will assume the accelerations $\ddot{\beta}$ as constant during τ:

$$\ddot{\beta}(t_2) = \ddot{\beta}(t_1). \tag{22}$$

Let us now write the performance index 19 for $t = t_2$ and introduce expressions 21 and 22. Now we will have:

$$\begin{aligned} J_i(t_2) = c_{oi}[\beta_i(t_1) + \dot{\beta}_i(t_1)\tau - \beta_i^o(t_2)] + c_{1i}[\dot{\beta}_i(t_1) + \ddot{\beta}_i(t_1)\tau \\ - \dot{\beta}^o(t_2)] + c_{2i}[\ddot{\beta}_i(t_1) - \ddot{\beta}^o(t_2)] = 0. \end{aligned} \tag{23}$$

Starting from the supposition that the ideal synergy is known, i.e. the laws of change in $\beta^o(t)$, $\dot{\beta}^o(t)$, and $\ddot{\beta}^o(t)$, and that the values of the phase coordinates can be measured by means of some transducers, the only unknown value in the expression 23 will be $\ddot{\beta}(t_1)$.

Solving expression 23 for $\ddot{\beta}(t_1)$, we find the acceleration values which must be applied to the locomotion system elements, in order to reduce the performance index 19 to zero after time interval τ:

$$\begin{aligned} \ddot{\beta}_i(t_1) = [c_{2i}\ddot{\beta}_i^o(t_2) - c_{oi}(\beta_i(t_1) + \dot{\beta}_i(t_1)\tau - \beta_i^o(t_2)) \\ -c_{1i}(\dot{\beta}_i(t_1) - \dot{\beta}^o(t_2)]/(c_{1i}\tau + c_{2i}). \end{aligned} \tag{24}$$

The nominator in expression 24 is evidently always positive and different from zero.

The compensating system cannot change the external synergy directly, but only indirectly by means of the internal synergy. For that reason let us find the relation between $\ddot{\varphi}_i(t_1)$ and $\ddot{\beta}_i(t_1)$. Since the values of $\ddot{\varphi}$ and $\ddot{\beta}$ are considered in the same time interval, the arguments can be left out in further elaboration.

For the purpose of simplifying the denotation, let us extend the φ vector with $(n + 1)$ component ξ and denote the new vector by φ^*

$$\varphi^* = (\varphi_1, \varphi_2, \ldots, \varphi_n, \xi).$$

It is evident that for a locomotion structure the relation between angles ϕ^* and β can be presented in the form:

$$\beta = B\varphi^* + \text{const.} \tag{25}$$

Since matrix B is not a function of time, differentiating twice by time we get:

$$\ddot{\varphi}^* = B^{-1}\ddot{\beta}. \tag{26}$$

Now the problem is to establish the torque values needed at each actuator, to ensure the demanded accelerations $\ddot{\varphi}_i^*$ to the system. The relation between the torques M_i and accelerations $\ddot{\varphi}_j^*$ can be obtained from the differential equations of the locomotion system, which for this purpose can be presented in a general form:

$$\mathrm{M}_i = \sum_{j=1}^{n+1} a_{ij}\ddot{\varphi}_j^* + f(\varphi^*, \dot{\varphi}^*), \tag{27}$$

where M_i = moments produced by actuators,

a_{ij} = a coefficient, depending on values $\varphi_i^*(i = 1, \ldots, n + 1)$.

These equations can be obtained analogously to Eqs. 12 and 13; the only difference is that the former are formed based on dynamic equilibrium around the centers of all the joints, and not around the fixed zero-moment point which, in fact, represents the first joint of the anthropomorphic mechanism. The advantage of expressing the dynamic equations in terms of the internal synergy is in the fact that the complete system regulation is done by changing the internal synergy and appropriate driving torques, as well.

The method of obtaining these equations and the computing algorithm, as well are considered in Ref. 9. In total we shall have n differential equations of the second order with n corresponding driving torques. It is important to underline that the $(n + 1)$-th coordinate $\varphi_{n+1}^* = \xi$ is "non-controllable" since it does not possess the drive. Due to this, stabilization of φ_{n+1}^* poses the basic problem in the regulation of perturbed regimes of motion of the anthropomorphic mechanisms.

From the system 27 the partial derivatives are:

$$\frac{\partial \mathrm{M}}{\partial \varphi^*} = a_{ij}. \tag{28}$$

By the matrix $A = \|a_{ij}\|$ let us denominate the sensitivity matrix. Let us write the relations between the driving torques and the angular accelerations needed, using the stated matrix:

$$\Delta \mathrm{M} = A^{-1} \Delta \ddot{\varphi}^*. \tag{29}$$

Introducing expression for 26 into 29 we get:

$$\Delta \mathrm{M} = A^{-1} B^{-1} \Delta \ddot{\beta}, \tag{30}$$

where ΔM—vector of compensating moments.

In this way the compensating actions are defined such that they are able to diminish the performance index 20 to zero during time interval τ, exact to second-order small values.

The choice of the time interval τ is to a certain extent arbitrary. It cannot be too large, because in that case supposition 22 does not hold. However, interval τ also cannot be too small, because in that case the compensating moment ΔM can grow to unacceptable values. Basically the value of τ can be understood as an adjustable system parameter, and its choice should be effected experimentally from the contradictory demands of system compensation quality and power demand.

It should be noted here, that the choice of interval τ does not mean a discrete realization of the compensation with step τ. The delay of the compensating system is being defined by a totally different value—the information processing time, according to the described method.

To avoid the application of a gyroscope, which the proposed method of stabilization requires, the regulation method by means of force measurement at the contact surface can be utilized in the cases of small perturbations.

Expressions for reaction forces at the point of contact (ZMP) between the extremity and support can be written (Fig. 3):

$$F_x = \sum_{i=1}^{11} m_i \ddot{x}_i = \ddot{\vartheta} \sum_{i=1}^{11} m_i V_i + \ddot{\psi} \sum_{i=1}^{11} m_i W_i + \sum_{i=1}^{11} m_i P_i, \tag{31}$$

$$F_y = \sum_{i=1}^{11} m_i \ddot{y}_i = \ddot{\vartheta} \sum_{i=1}^{11} m_i A_i + \sum_{i=1}^{11} m_i C_i, \tag{32}$$

$$F_z = \sum_{i=1}^{11} m_i \ddot{z}_i + \sum_{i=1}^{11} m_i g = \ddot{\vartheta} \sum_{i=1}^{11} m_i R_i + \ddot{\psi} \sum_{i=1}^{11} m_i S_i + \sum_{i=1}^{11} m_i T_i + \sum_{i=1}^{11} m_i g, \tag{33}$$

where V_i, W_i, P_i, A_i, C_i, R_i, S_i and T_i are functions of the set prescribed $(\beta_i, \dot{\beta}_i, \ddot{\beta}_i)$ and of the computed synergy $(\psi, \vartheta, \dot{\psi}, \dot{\vartheta})$. It is evident that in that case there exists some relation between the force-vector:

$$F = \{F_x, F_y, F_z\},$$

and the corresponding dynamic moments round three axes are:

$$M = \{M_x, M_y, M_z\}.$$

It should be pointed out that a possibility exists to express the reaction forces by means of analytical expressions, which can be symbolically written in the form:

$$\begin{aligned} F_x &= f_1(\ddot{\psi}, \ddot{\vartheta}, \dot{\psi}, \dot{\vartheta}, \psi, \vartheta, \ddot{\beta}, \dot{\beta}, \beta), \\ F_y &= f_2(\ddot{\psi}, \ddot{\vartheta}, \dot{\psi}, \dot{\vartheta}, \psi, \vartheta, \ddot{\beta}, \dot{\beta}, \beta), \\ F_z &= f_3(\ddot{\psi}, \ddot{\vartheta}, \dot{\psi}, \dot{\vartheta}, \psi, \vartheta, \ddot{\beta}, \dot{\beta}, \beta). \end{aligned} \tag{34}$$

Knowing that the components of reaction forces F_x and F_y affect only the moment M_z, and supposing that the latter mainly influences the direction of motion of the locomotion structure, we restrict ourselves to the observation of the vertical reaction components F_z only.

It is also supposed that the "internal" algorithm* of the locomotion system is being realized sufficiently exactly and that perturbations appear only in the external coordinates of the system.

Let the transducers, measuring the vertical reaction force be arranged as shown in Fig. 9. According to this scheme, the moment equations due to the vertical components of the reaction forces in relation to the nominal position point of the resulting force, can be written as follows:

$$\begin{aligned} s(F_{Bz} - F_{Cz}) &= M_x, \\ d_1 F_{Az} - d_2(F_{Bz} + F_{Cz}) &= M_y. \end{aligned} \tag{35}$$

where F_{Az}, B_{Bz}, and F_{Cz} are the measured values of the vertical reaction components. On the basis of 35, it is clear that M_x and M_y in the nominal regime reduce to zero. Due to perturbation of the vertical reaction components F_z, some disturbing moments ΔM_x and ΔM_y result. If we suppose that the perturbations are small and start from the fact that accelerations are the most sensitive values, directly interconnected with the perturbing moments generated, the following generally based relations can be formed:

$$\begin{aligned} M_x &= M_x(\ddot{\psi}, \ddot{\vartheta}, \dot{\psi}, \dot{\vartheta}, \psi, \vartheta, \ddot{\beta}_i, \dot{\beta}_i, \beta_i), \\ M_y &= M_y(\ddot{\psi}, \ddot{\vartheta}, \dot{\psi}, \dot{\vartheta}, \psi, \vartheta, \ddot{\beta}_i, \dot{\beta}_i, \beta_i). \end{aligned} \tag{36}$$

* "Internal" algorithm reveals the angles of the kinematic-dynamic algorithm, realized by appropriate drive systems.

where β_i are coordinates of the kinematic algorithm (Fig. 3)

$$\begin{aligned}\Delta M_x &\approx \frac{\partial M_x}{\partial \ddot{\psi}}\Delta\ddot{\psi} + \frac{\partial M_x}{\partial \ddot{\vartheta}}\Delta\ddot{\vartheta} + \sum_{i=1}^{3}\frac{\partial M_x}{\partial \ddot{\beta}_i}\Delta\ddot{\beta},\\ \Delta M_y &\approx \frac{\partial M_y}{\partial \ddot{\psi}}\Delta\ddot{\psi} + \frac{\partial M_y}{\partial \ddot{\vartheta}}\Delta\ddot{\vartheta} + \sum_{i=1}^{3}\frac{\partial M_y}{\partial \ddot{\beta}_i}\Delta\ddot{\beta}.\end{aligned} \tag{37}$$

At this point it is supposed that all components of the kinematic-dynamic algorithm vector are constant, so that the partial derivatives in the system 37 are of the type:

$$\frac{\partial M_x}{\partial \ddot{\psi}},\ \frac{\partial M_x}{\partial \ddot{\vartheta}},\ \frac{\partial M_x}{\partial \ddot{\beta}},\ \frac{\partial M_y}{\partial \ddot{\psi}},\ \frac{\partial M_y}{\partial \ddot{\vartheta}},\ \frac{\partial M_y}{\partial \ddot{\beta}}\Big/\dot{\psi},\ \dot{\vartheta},\ \psi,\ \theta,\ \dot{\beta},\ \beta = \text{const.}$$

It should be underlined that this is a realistic supposition, if we keep in mind the fact that the perturbations are small, as well as the fact, that the acceleration changes are a direct consequence of the corresponding change in the perturbing forces.

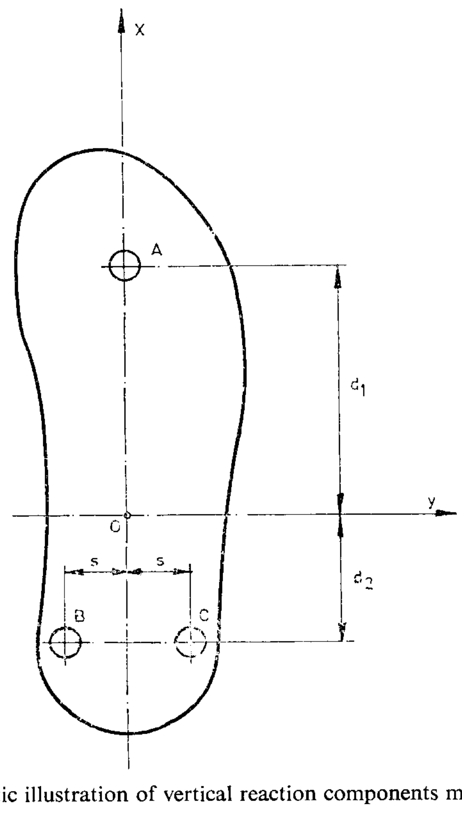

FIG. 9. Schematic illustration of vertical reaction components measurement.

Due to the supposition of the constant "internal" algorithm of the locomotion structure, it is most natural to assume that the increments of all accelerations of its elements in one plane are equal, so that:

$$\Delta\ddot{\psi} = \Delta\ddot{\beta}_i.$$

In that case, by solving system 37 for acceleration increments in the sagittal and frontal planes, we will have:

$$\Delta\ddot{\psi} = \frac{\Delta_1}{\Delta}, \qquad \Delta\ddot{\vartheta} = \frac{\Delta_2}{\Delta}, \tag{38}$$

where

$$\Delta_1 = \Delta M_y \frac{\partial M_x}{\partial\ddot{\vartheta}} - \Delta M_x \frac{\partial M_y}{\partial\ddot{\vartheta}},$$

$$\Delta_2 = \Delta M_y\left(\frac{\partial M_x}{\partial\ddot{\psi}} + \sum_{i=1}^{3}\frac{\partial M_x}{\partial\ddot{\beta}_i}\right) - \Delta M_x\left(\frac{\partial M_y}{\partial\ddot{\psi}} + \sum_{i=1}^{3}\frac{\partial M_y}{\partial\ddot{\beta}_i}\right),$$

$$\Delta = \left(\frac{\partial M_x}{\partial\ddot{\psi}} + \sum_{i=1}^{3}\frac{\partial M_x}{\partial\ddot{\beta}_i}\right)\frac{\partial M_y}{\partial\ddot{\vartheta}} - \left(\frac{\partial M_y}{\partial\ddot{\psi}} + \sum_{i=1}^{3}\frac{\partial M_y}{\partial\ddot{\beta}_i}\right)\frac{\partial M_x}{\partial\ddot{\vartheta}}. \tag{39}$$

Keeping in mind the relations 27, which connect the accelerations of the "internal" coordinates of the kinematic-dynamic program with the corresponding driving moments, as well as the connection between the "internal" and "external" coordinates (Eq. 25), we can apply again relation 30 to calculate the compensating moments.*

It has to be pointed out that the regulation method of force measurement is applicable while all three pressure transducers are under load. This means that at some greater perturbation, when the foot starts to separate from the support, the gyroscope version finds its application.

The procedure described can ensure gait stability in the presence of small perturbations. As far as internal synergy is concerned, the supposition of small deviations from the ideal synergy can be considered justified. At gait upon known terrain, the external synergy also has no reason to change much. However, all external factors cannot be known in every detail; consequently, the external synergy may undergo deviations which cannot be regarded as small. The attempt to compensate such deviations only by means of the method displayed, may lead to the opposite effect. In that case the compensating moments will be relatively great and in the case of greater deviations the nonlinearities of the system may become of deciding influence.

In the case of greater deviations, it is much more appropriate not to try to compensate for them in one step, but to pass to a new ideal internal

* In this compensation method there exist only two increment types of accelerations of the system: $\Delta\ddot{\psi}$ and $\Delta\ddot{\vartheta}$.

synergy in the course of a time interval and then gradually return to the old one. According to this approach, for the compensation of such deviations it is necessary to apply the first control method, from which a new ideal synergy is chosen.

Let β^o be the ideal synergy in the absence of perturbations. For the sake of stability maintaining in the case of great deviations let us temporarily choose another ideal synergy, which we will denote with β^*.

Let us suppose the existence of a family of external synergies β^* depending on the parameter vector p

$$\beta^*(p)$$
$$p = (p_1, p_2, \ldots, p_m)' \tag{40}$$

where m is the number of system parameters.

The permissible values of the parameters can be geometrically represented as some region of a m-dimensional parametric space. Let us denote this region the working region. To each point of this working region there corresponds a certain vector function β^*. The values of β^* should be chosen in such a way, that in the first place a greatest possible approach to the real synergy β is effected, and then to ensure a gradual approach to the original ideal synergy β^o.

First the synergy is assessed, which is nearest to the real synergy β, from the family 40. Let us denote this synergy with β^* and write a criterion of the form:

$$J_i = \tfrac{1}{2}[c_{0i}(\beta_i - \beta_i^A) + c_{1i}(\dot{\beta}_i - \dot{\beta}_i^A)]^2 \qquad (i = 1, \ldots, n). \tag{41}$$

Now let us find the minimum of the expressions 41:

$$J_m = \min_p J_\Sigma, \tag{42}$$

where

$$J_\Sigma = \sum_{i=1}^{n} J_i.$$

Let us form the expressions of the partial derivatives of J_Σ with respect to parameters:

$$\begin{aligned} \frac{\partial J_\Sigma}{\partial p_j} &= \sum_{i=1}^{n} \frac{\partial J_i}{\partial p_j}, \\ \frac{\partial J_i}{\partial p_j} &= -[c_{oi}(\beta_i - \beta_i^A) + c_{1i}(\dot{\beta}_i - \dot{\beta}_i^A)]\left(c_{oi}\frac{\partial \beta_i^A}{\partial p_j} + c_{1i}\frac{\partial \dot{\beta}_i^A}{\partial p_j}\right). \end{aligned} \tag{43}$$

The condition of minimum is represented by:

$$\frac{\partial J_\Sigma}{\partial p_j} = 0, \qquad (j = 1, 2, \ldots, m). \tag{44}$$

Let us denote the value of parameter p obtained from 44 with p^A. However, let us choose the working point p^* not in the point p^A, but in another point, translated in the sense of p^o of the working region:

$$p^* = p^A + \lambda(p^o - p^A), \qquad 0 < \lambda < 1. \tag{45}$$

By means of λ we are able to ensure a gradual approach to the synergy β^o.

In the described method, the most difficult task is to define point p^A. As the vector β has many components and is function of several parameters and of time, it is impossible to store all variants in the control computer memory. So for the sake of a practical realization of the method described, it is necessary to find an analytical approximation of the functions of the ideal synergies on parameters and time. By studying various human gaits, some characteristics of the synergies were noted. They can be represented sufficiently precisely by a two-parametric family of curves. These parameters are the following:

S—step length parameter

T—gait speed parameter.

Based upon these parameters, sufficiently simple approximations can be formed. Let β^o be some synergy (gait) chosen as the basic synergy. Now we can represent the mentioned family of synergies in the following form (let us denote the lower part of the locomotion system by the index d):

$$\beta_d(t) = S\beta_d^0\left(\frac{t}{T}\right). \tag{46}$$

As explained earlier for the upper part of the system the synergy was defined from the repeatability conditions 14. By varying the values of S and T we can acquire a series of graphs, which illustrate the upper locomotion system synergy change as a function of these parameters. Some of these graphs are illustrated in Fig. 10.

The analysis of these graphs shows that for the system upper part the relation between β_{up} and p can be approximately written in the form:

$$\beta_{up} = \beta_{up}^o + A_s(S - S^o) + A_T(T - T^o), \tag{47}$$

where A_s and A_T are vectors, whose components are functions of time:

$$A_s = \left\{\frac{\partial\beta_{up,i}^o}{\partial S}\right\}, \qquad A_T = \left\{\frac{\partial\beta_{up,i}^o}{\partial T}\right\}. \tag{48}$$

It is necessary now to find the partial derivatives $\partial\beta_i/\partial S$ and $\partial\beta_i/\partial T$. For the system lower part we will have:

$$\frac{\partial\beta_{d,i}}{\partial S} = \beta_i^o\left(\frac{t}{T^o}\right), \tag{49}$$

$$\frac{\partial\beta_{d,i}}{\partial T} = -\frac{tS^o}{(T^o)^2}\dot{\beta}^o\left(\frac{t}{T^o}\right). \tag{50}$$

For the system upper part these derivatives can be written (48):

$$\frac{\partial \beta_{up,i}}{\partial S} = A_s^i, \qquad \frac{\partial \beta_{up,i}}{\partial T} = A_T^i. \tag{51}$$

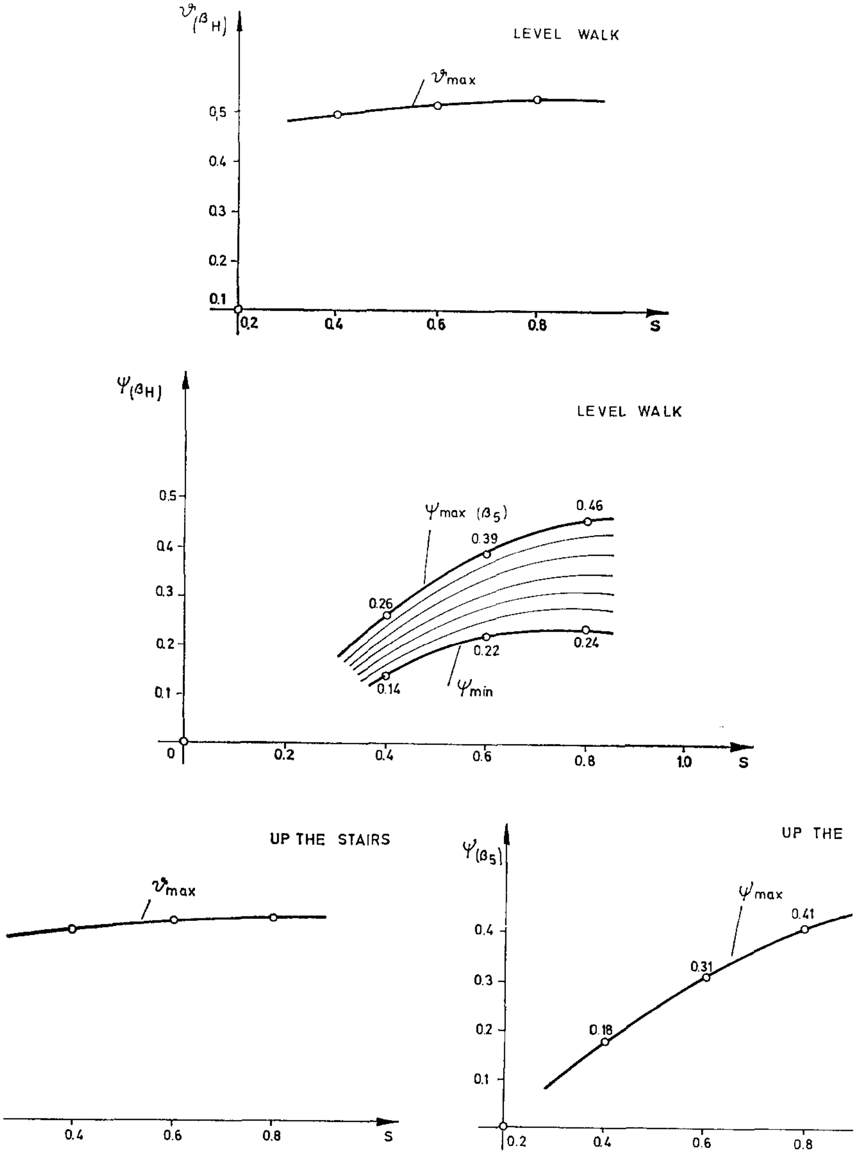

FIG. 10. Upper part synergy in dependence on characteristic parameters.

Let us augment the matrices A_S and A_T by introducing into them the components, corresponding to the upper part of the model:

$$B_S^j = \begin{cases} \beta_j^o\left(\dfrac{t}{T^o}\right) & \text{—if } \beta_j \text{ relates to the lower part} \\ A_S^j & \text{—if } \beta_j \text{ relates to the upper part} \end{cases}$$

$$B_T^j = \begin{cases} \dfrac{tS^o}{(T^o)^2}\beta_j^o\left(\dfrac{t}{T^o}\right) & \text{—if } \beta_j \text{ relates to the power part} \\ A_T^j & \text{—if } \beta_j \text{ relates to the upper part.} \end{cases}$$

Now the arbitrary synergy can be written as:

$$\beta = \beta^o + B_S(S - S^o) + B_T(T - T^o). \tag{52}$$

Introducing this expressions into 43 instead of β^A we get:

$$\begin{aligned} \frac{\partial J_i}{\partial S} &= -[c_{oi}(\beta_i - \beta_i^o - B_{Si}(S - S^o) - B_{Ti}(T - T^o)) \\ &\quad + c_{1i}(\dot{\beta}_i - \dot{\beta}_i^o - \dot{B}_{Si}(S - S^o) - \dot{B}_{Ti}(T - T^o))](c_{oi}B_{Si} + c_{1i}\dot{B}_{Si}), \\ \frac{\partial J_i}{\partial T} &= -[\ldots](c_{oi}B_{Ti} + c_{1i}\dot{B}_{Ti}). \end{aligned} \tag{53}$$

By summing expressions 53 with respect to i and putting the results equal to zero according to relation 44 we get:

$$\begin{aligned} a_{11}S + a_{12}T &= b_1, \\ a_{21}S + a_{22}T &= b_2, \end{aligned} \tag{54}$$

where

$$\begin{aligned} a_{11} &= \sum_{i=1}^{n} (c_{oi}B_{Si} + c_{1i}\dot{B}_{Si})^2, \\ a_{12} &= \sum_{i=1}^{n} (c_{oi}B_{Ti} + c_{1i}\dot{B}_{Ti})(c_{oi}B_{Si} + c_{1i}\dot{B}_{Si}), \\ a_{21} &= a_{12}, \qquad a_{22} = a_{11}, \end{aligned} \tag{55}$$

$$\begin{aligned} b_1 &= \sum_{i=1}^{n} [c_{oi}(\beta_i - \beta_i^o + B_{Si}S^o + B_{Ti}T^o) \\ &\quad + c_{1i}(\dot{\beta}_i - \dot{\beta}_i^o + \dot{B}_{Si}S^o + \dot{B}_{Ti}T^o)](c_{oi}B_{Si} + c_{1i}\dot{B}_{Si}), \\ b_2 &= \sum_{i=1}^{n} [\ldots](c_{oi}B_{Ti} + c_{1i}\dot{B}_{Ti}). \end{aligned} \tag{56}$$

The system of Eqs. 54 makes it possible to acquire values of S and T minimizing 44, i.e. to obtain point "A" in the working region. Point p^* (S^*, T^*) is obtained by means of 45.

The system determinant 54 is always greater than zero:

$$\Delta = a_{11}^4 + a_{22}^4 + a_{12}^2 \cdot a_{21}^2$$

so that it is always possible to obtain the values of S and T

$$S = \frac{b_1 a_{22} - b_2 a_{12}}{\Delta}, \qquad T = \frac{a_{11} b_2 - a_{21} b_1}{\Delta}. \tag{57}$$

Now it is possible to make one step more in the simplification of the described method.

It should be remembered that the coefficients of Eq. 54, which define the necessary change of parameters, are time dependent. Therefore, for a practical realization of such a method it is necessary to store in the control computer memory the matrices $B_S(t)$ and $B_T(t)$ calculated for various moments of time. In Fig. 11 there are given the graphs of ideal synergy for the upper and lower part of the locomotion system.

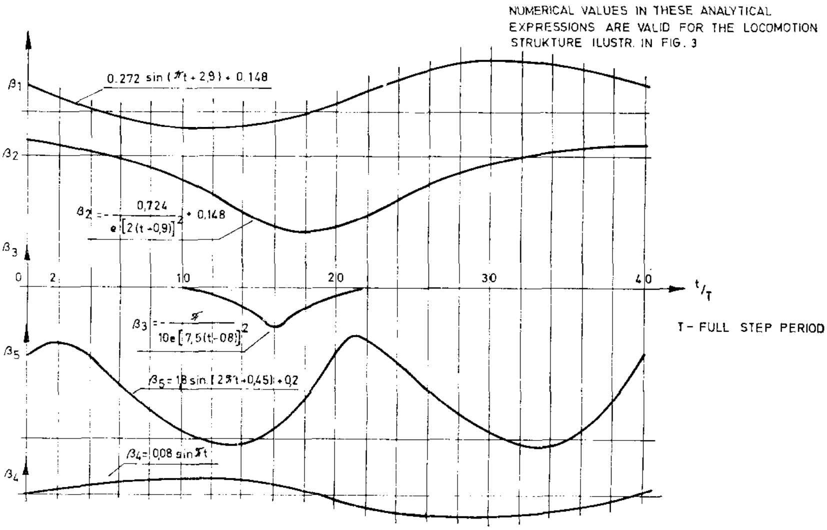

FIG. 11. Analytic presentation of locomotion external synergy.

These graphs show that the ideal synergy of the model β^o does change sufficiently smoothly, so that an acceptable analytical approximation can be adopted for its presentation:

$$\begin{aligned}
\beta_1(t) &= A \sin(\pi t + \varphi_o) + A_o, \\
\beta_2(t) &= -\, B/e^{[2(t-t_o)]^2} + B_o, \\
\beta_3(t) &= -\, C/e^{[a(t-t_o)]^2}, \\
\beta_4(t) &= D \sin \pi t; \quad \beta_4 = \vartheta \\
\beta_5(t) &= E \sin(2\pi t + \varphi^o) + E_0; \quad \beta_5 = \psi.
\end{aligned} \tag{58}$$

β_1, β_2, β_3 represent the lower part synergy of the locomotion system, β_4 and β_5 represent its upper part synergy.

Placing these relations into expressions 47, 51, 55 and 56 we will obtain the relations of coefficients of the equations 54 as functions of time and after that, by means of 57 the required values of S and T can be acquired.

In the case under consideration, $\beta_i(t)$ have not been too complicated, so that a relatively simple approximation is allowable. However, with some other gait types or gait upon complex profiles, the $\beta_i(t)$ coordinates may become complex, so that a sufficiently precise approximation cannot be found in that case. As already stated, a great number of $B_S(t)$ and $B_T(t)$ matrices must be memorized, complicating the already delicate problem concerning the demands in the realization of the specialized, miniaturized control computer.

For the cases, another method of ideal synergy choice is proposed here. It is based on the supposition that the greatest deviations from the ideal synergy (kinematic-dynamic program) occur in the upper part of the locomotion system. Accordingly, the ideal synergy will be chosen from the conditions of compensation of the system upper-part deviations.

Let us consider the phase space vector Y:

$$Y = \begin{Bmatrix} \beta_4 \\ \beta_5 \\ \dot{\beta}_4 \\ \dot{\beta}_5 \end{Bmatrix}. \tag{59}$$

The values of Y at the beginning and end of half period let us denote with Y^o and Y^T. The phase vectors values in the case of ideal synergy let us denote by a dash above the symbol.

The repeatability conditions for upper part of the system can be written in the form:

$$\bar{Y}^o + \Delta Y^o = \eta(\bar{Y}^T + \Delta Y^T), \tag{60}$$

where

$$\eta = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

The phase vector deviation at the end of the half-period can be expressed as:

$$\Delta Y^T = \left(\frac{\partial Y^T}{\partial p_1}\right)\Delta p_1 + \left(\frac{\partial Y^T}{\partial p_2}\right)\Delta p_2 + \dots + \left(\frac{\partial Y^T}{\partial p_s}\right)\Delta p_s \tag{61}$$

s is the number of parameters.

By simultaneously solving 60 and 61 we get [7]

$$D\Delta p - d = 0 \tag{62}$$

$$D = \begin{bmatrix} -\dfrac{\partial Y_1^T}{\partial p_1} & -\dfrac{\partial Y_1^T}{\partial p_2} & \cdots & -\dfrac{\partial Y_1^T}{\partial p_s} \\ \dfrac{\partial Y_2^T}{\partial p_1} & \dfrac{\partial Y_2^T}{\partial p_2} & \cdots & \dfrac{\partial Y_2^T}{\partial p_s} \\ -\dfrac{\partial Y_3^T}{\partial p_1} & -\dfrac{\partial Y_3^T}{\partial p_2} & \cdots & -\dfrac{\partial Y_3^T}{\partial p_s} \\ \dfrac{\partial Y_4^T}{\partial p_1} & \dfrac{\partial Y_4^T}{\partial p_2} & \cdots & \dfrac{\partial Y_4^T}{\partial p_s} \end{bmatrix} \qquad d = \begin{Bmatrix} \bar{Y}_1^o + \Delta Y_1^o + \bar{Y}_1^T \\ \bar{Y}_2^o + \Delta Y_2^o - \bar{Y}_2^T \\ \bar{Y}_3^o + \Delta Y_3^o + \bar{Y}_3^T \\ \bar{Y}_4^o + \Delta Y_4^o - \bar{Y}_4^T \end{Bmatrix}. \tag{63}$$

If the number of parameters s is equal to the number of phase coordinates from Eqs. 62, the values of Δp can be acquired directly. If, however, the number of parameters is smaller, condition 62 cannot be satisfied exactly. In this case let us introduce the performance index of the form [11]:

$$J = (D\Delta p - d)'(D\Delta p - d). \tag{64}$$

The increment of parameter Δp can be obtained from the condition of minimum of the expression 64:

$$\frac{\partial J}{\partial(\Delta p)} = 0,$$

wherefrom

$$\Delta p = (D'D)^{-1}D'd. \tag{65}$$

Now we can obtain the parameter values, at which the ideal synergy will be closest to the real one. Earlier (see 45) these parameter values we denoted by p^A

$$p^A = p_o + \Delta p. \tag{66}$$

In the case that the first deviations have been too great, it is good practice to accomplish a few approximation steps and use, instead of 66, the relation:

$$p_i^A = p_{i-1}^A + \varepsilon \Delta p_i, \tag{67}$$

where i is the number of iterations; and ε is the positive multiplier ($0 < \varepsilon \leqslant 1$), and is introduced for improving the iterative process convergence.

STABILITY MARGIN OF ANTHROPOMORPHIC SYSTEM

A two-legged locomotion system possesses, due to its geometrical features, a certain stability margin. This makes possible the automatic compensation of small perturbations. We are going to illustrate this fact on the simplest model of the system (Fig. 12).

The model is represented by a body of mass m and inertial moment J

connected with a massless "foot". Let us suppose that for some reasons the "foot" is declined in relation to the ground for an angle ξ. In the case of sufficiently small perturbations ξ the system will return to the position of static equilibrium.

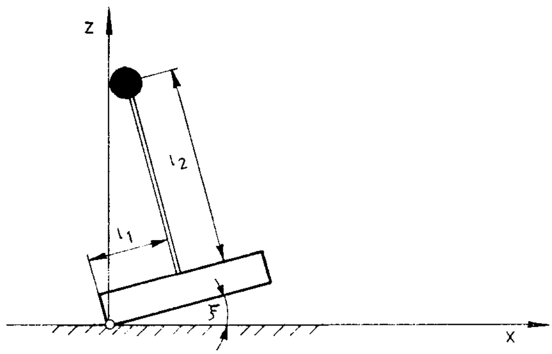

FIG. 12. Simplest system with foot.

An analogue case appears with the two-legged locomotion systems. The "foot" in this case is the foot of the leg, ensuring the model some stability margin, in the function of its surface. With some suppositions to be discussed later, the locomotion system stability margin obtaining can be reduced to the analysis of the simplest system, illustrated in Fig. 12.

Let us suppose that internal synergy $\varphi(t)$ is being realized exactly and that the perturbation effects the model as a whole, "balancing" on the supporting foot, shifting the support from one foot edge to the other. Such a supposition is fully justified, if the maintenance of the internal synergy is ensured by special tracking systems. In that case the errors $\Delta\varphi$ will be sensibly smaller than the perturbations of the external synergy. In such a case the perturbations can be represented by angles, formed by the foot and the support surface, and the angular rates of these angles.

The locomotion model can "balance" on the supporting foot in two planes: the sagittal and frontal ones. (Turning around the z-axis must not be considered, as this does not effect loss of stability, but merely the change of motion direction). From the stability standpoint the most dangerous perturbations are in the frontal plane, as the dimension of the foot is much smaller in this plane compared to its dimension in the sagittal plane. Besides, the deviations of the model movement in the sagittal plane can be to a certain extent limited by the presence of the other foot, coming in contact with the support plane earlier or later, depending on the perturbations. In the frontal plane, such a limitation (supporting) is in one of the possible motion directions excluded.

For this reason let us examine the stability margin in the frontal plane. However, the displayed approach can be broadened without difficulties to the case of deviations in two planes.

With these suppositions, the stability margin analysis can be reduced to the analysis of the planar model from Fig. 12.

Let us first consider the case of perturbed motion on the basis of the ideal synergy, obtained from the conditions of the locomotion system dynamic equilibrium.[1] This case can be of practical interest in very, very slow gait. Let us suppose that mass m equals the total mass of the locomotion system. The length of the stick l_2 should be chosen in such a way that position of mass m in relation to the foot corresponds to the position of the locomotion-system center of gravity, and the moment of inertia J is equal to the reduced-system moment of inertia, in relation to the center of gravity.

The internal synergy $\varphi(t)$ being known, the position of the center of gravity can be calculated at all times, i.e. the value l_2 can be calculated.

Let us write the expression for potential energy:

$$E_p = mgz_m, \tag{68}$$

where z_m is the vertical coordinate of mass m. It follows:

$$z_m = l_2 \sin(\alpha + \xi) + l_1 \sin \xi \operatorname{sign} \zeta. \tag{69}$$

In Fig. 12, a few graphs are presented which represent the dependance of the potential energy of the model from the angle ξ, for various values of the length l_2, corresponding to various instants of the imposed kinematic program. The critical values of angles ξ for $\xi > 0$ and $\xi < 0$ let us denote by ξ^*. They can be obtained from expression 69.

Let us now write the expression for the kinetic energy of the model:

$$E_k = \tfrac{1}{2} J_o \dot{\xi}^2, \tag{70}$$

where J_o is the reduced moment of inertia with respect to the point of support.

$$J_o = J + m(x_m^2 + z_m^2). \tag{71}$$

The potential energy values in the critical points let us denote by E_p^*.

The condition for system stability will be now:

$$E_k + E_p \leqslant E_p^*. \tag{72}$$

From the graph in Fig. 13 and the relation 72 the speed $\dot{\xi}$ can be obtained for every value of angle ξ for which the model will preserve its stability. Let us dispose of some value of angle $\xi = \xi_1$. From the graph, Fig. 13, we can obtain the corresponding potential energy value E_{p1}. In order

[1] It is supposed to be sufficiently accurate to take "frozen" coefficients that correspond to different times of the "legs" algorithm supposed.

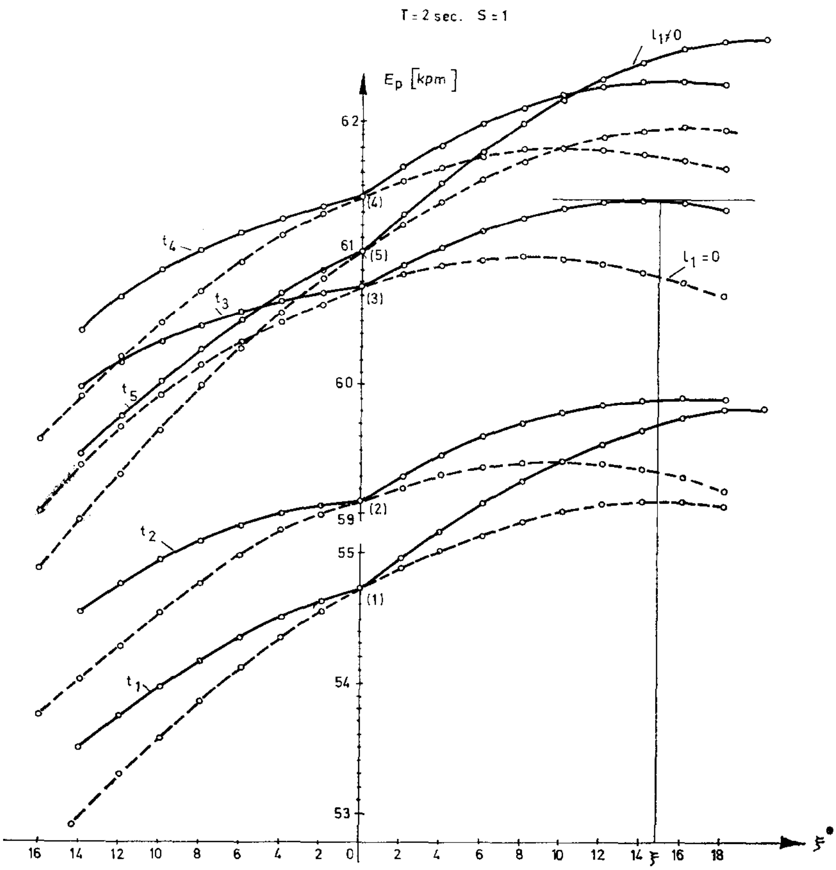

FIG. 13. Stability margin presentation.

for the model to be stable, the kinematic energy margin E_{k1} is needed:

$$E_{k1} \leqslant E_p^* - E_{p1}. \tag{73}$$

Introducing now the expression for the kinetic energy 70 the admissible value of speed $\dot{\xi}$ can be obtained:

$$\dot{\xi}_1^2 \leqslant \frac{2}{J_0}(E_p^* - E_{p1}). \tag{74}$$

In this way, to every value of angle ξ there corresponds some critical value of $\dot{\xi}$. All admissible values ξ and $\dot{\xi}$ form a closed region of the phase plane, as shown in Fig. 14. This region defines a stability margin corresponding to the case of a very slow gait. If the perturbations do not leave this region, they will be automatically compensated. In the cases, in which the point corresponds to the dashed region, it is indispensable to engage the compensation system, based upon one of the described methods. In fact, the compensation system should be engaged earlier, in order to dispose with a certain stability margin coefficient. Such case of security in the disposable stability margin is illustrated in Fig. 14 by the dashed line.

In the cases, when the transient process is insufficiently faster than the basic algorithm motion, a simultaneous dynamic analysis of the model motion according to the prescribed synergy is needed, as well as the perturbed motion.

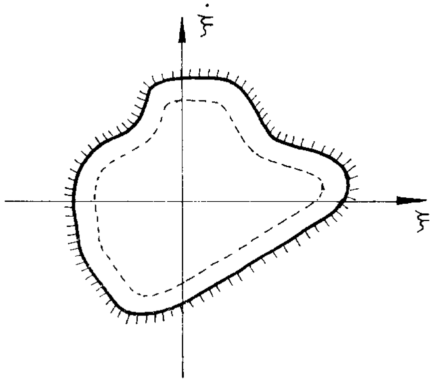

FIG. 14. Potential energy in dependendence on perturbation ξ.

Such analysis can also be reduced to the analysis of an equivalent model. For this reason let us make the model (Fig. 12) more complex, supposing that the stick is connected with the "foot" by means of a simple joint and that the relative angle α in that simple joint changes according to a known law (Fig. 15). This law is defined by the ideal synergy and α is obtained from the condition that the position of mass m relative to the

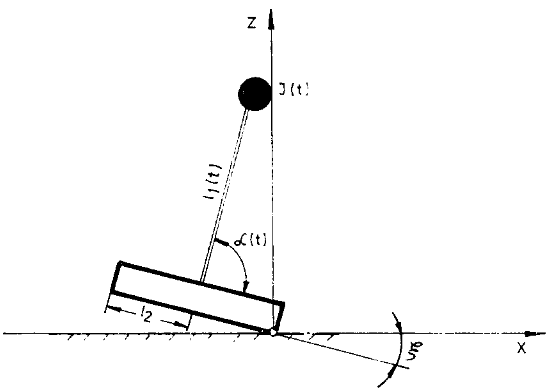

FIG. 15. Equivalent biped model.

foot corresponds to the position of the mass center of the locomotion system relative to the supporting surface of the foot.

As the internal synergy is known, it is possible to calculate the position of the center of system gravity in every moment, expressed in coordinates z', y' (Fig. 15).

$$y'_c = \frac{1}{M}\sum_{i=1}^{n} m_i y'_i, \qquad z'_c = \frac{1}{M}\sum_{i=1}^{n} m_i z'_i, \tag{75}$$

where n is the number of the locomotion system elements

M is the total mass of the system.

Knowing y'_c and z'_c we can obtain values of l_2 and α for the equivalent model:

$$\begin{aligned} \alpha_r &= \operatorname{arc\,tan} \frac{z'_c}{y'_c}, \\ l_2 &= \sqrt{(y'^2_c + z'^2_c)}. \end{aligned} \tag{76}$$

Equivalent moment of inertia can be obtained from the moment equation

$$J_r\ddot{\alpha}_r - m\ddot{y}_c z_c - (\ddot{z}_c + g)y_c = 0,$$

wherefrom

$$J_r = \frac{m}{\ddot{\alpha}_r}[\ddot{y}_c z_c + (\ddot{z}_c + g)y_c]. \tag{77}$$

Let us write now the equations of motion for the model from Fig. 15. The coordinates of the mass center are:

$$\begin{aligned} y_m &= ql_1 \cos\xi + l_r \cos(\alpha + \xi), \\ z_m &= ql_1 \sin\xi + l_r \sin(\alpha + \xi), \\ q &= \operatorname{sign}\xi. \end{aligned} \tag{78}$$

The angle ξ will be considered as negative when the model is supported on the right edge of the foot (as illustrated in Fig. 15).

By differentiating expression 78 twice, for every ξ except $\xi=0$, we get:

$$\begin{aligned} \ddot{y}_m &= -z_m + A_y,\ddot{\xi} \\ \ddot{z}_m &= y_m\ddot{\xi} + A_z, \end{aligned} \tag{79}$$

where

$$\begin{aligned} A_y = &-ql_1\dot{\xi}^2\cos\xi + \ddot{l}_2\cos(\xi + \alpha) - 2\dot{l}_r(\dot{\alpha} + \dot{\xi})\sin(\alpha + \xi) \\ &- l_r\ddot{\alpha}\sin(\alpha + \xi) - l_r(\dot{\alpha} + \dot{\xi})^2\cos(\alpha + \xi), \\ A_z = &-ql_1\dot{\xi}^2\sin\xi + \ddot{l}_r\sin(\xi + \alpha) + 2\dot{l}_r(\dot{\alpha} + \dot{\xi})\cos(\alpha + \xi) \\ &+ l_r\ddot{\alpha}\cos(\alpha + \xi) - l_r(\dot{\alpha} + \dot{\xi})^2\sin(\alpha + \xi). \end{aligned} \tag{80}$$

Now let us write the expressions for forces and inertial force moments:

$$\begin{aligned} F_y &= mz_m\ddot{\xi} - A_y m, \\ F_z &= -my_m\ddot{\xi} - mA_z, \\ M &= -J_r(\ddot{\xi} + \ddot{\alpha}). \end{aligned} \tag{81}$$

By putting the sum of external forces moments and the inertial forces equal to zero around the support point, we get the differential equations of motion in the form:

$$[J_r + m(z_m^2 + y_m^2)]\ddot{\xi} = m(zA_y - yA_z) - mgy_m - J_r\ddot{\alpha}_r. \qquad (82)$$

The values $\dot{l}_r$, $\ddot{l}_r$, $\dot{\alpha}$, $\ddot{\alpha}$ are obtained by differentiating 75 and 76.

The moment of inertia J_r in this method is assumed to change stepwise at the beginning of each step, and to stay constant during integration.

It should be noted that the values l_r, $\dot{l}_r$, $\ddot{l}_r$, α, $\dot{\alpha}$, $\ddot{\alpha}$, and J_r are obtained from the ideal synergy, and as this synergy is known, these values can be calculated for every moment of the program:

$$l_r = (y_c^2 + z_c^2)^{1/2},$$

$$\alpha_r = \arccos\left(\frac{y_c}{l_r}\right),$$

$$\dot{l}_r = \frac{z_c\dot{z}_c + y_c\dot{y}_c}{l^*},$$

$$\dot{\alpha}_r = \frac{\dot{l}_r \cos\alpha_r - \dot{y}_c}{l_r \sin\alpha_r},$$

$$\ddot{\alpha}_r = \frac{(\ddot{l}_r \cos\alpha_r - \dot{l}_r\dot{\alpha}_r \sin\alpha_r - \ddot{y}_c)l_r \sin\alpha_r}{l_r^2 \sin^2\alpha_r} - \frac{(\dot{l}_r \cos\alpha_r - \dot{y}_c)(\dot{l}_r \sin\alpha_r + l_r\dot{\alpha}_r \cos\alpha_r)}{l_r^2 \sin^2\alpha_r},$$

$$\ddot{l}_r = \frac{\dot{z}_c^2 + \dot{y}_c^2 + z_c\ddot{z}_c + y_c\ddot{y}_c - \dot{l}_r^2}{l_r}.$$

Accordingly, conditions for overturning will be (Fig. 16): For

$$\xi > 0 \qquad y_c \leqslant 0, \qquad \dot{y}_c < 0. \qquad (83)$$

For $\xi < 0$ the calculation is prolonged to the moment when the other leg

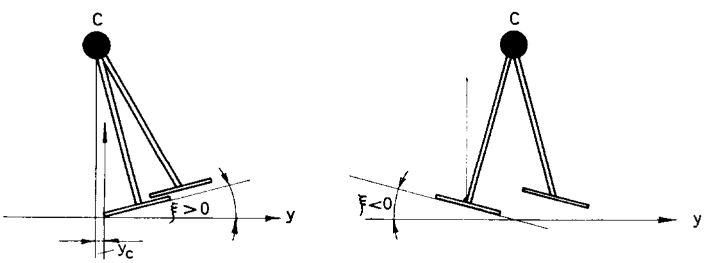

FIG. 16. Equivalent scheme of the criterion application for the obtaining of the dynamic stability margin.

of biped being—till then in the air—hits the ground. At the moment of contact the following conditions are imposed on the system:

$$(1)\ T^* = (1 - \delta_1)T,$$
$$(2)\ E_k^* = \tfrac{1}{2}J^*\dot{\xi}^2_{(T^*)} \leqslant \delta_2 E^*, \tag{84}$$

where T^*—represents the time of contact of the other leg,
δ_1—number given in advance,
J^*—moment of inertia of the equivalent system with respect to the contact point,
$\dot{\xi}_{(T^*)}$—velocity of perturbation at the moment of contact,
E^*—maximum value of potential energy for the time section corresponding to the time of accomplishing the contact, and
δ_2—number given in advance.

So from the set of initial conditions ξ_0, $\dot{\xi}_0$ we are interested only in solutions that correspond to the requirements 83, and 84, for $\xi > 0$ and $\xi < 0$, respectively.

Figure 17 illustrates the result of investigating the autonomous stability for the locomotion model shown in Fig. 3.

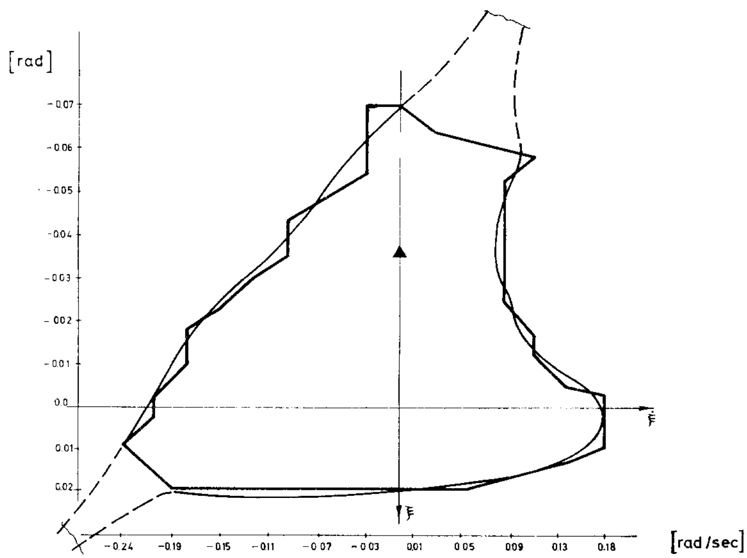

FIG. 17. Results of gait stability for the model in Fig. 3.

PRELIMINARY ORGANIZATION OF SPECIAL-PURPOSE COMPUTER FOR BIPED LOCOMOTION-SYSTEM CONTROL

The gait to be performed is described by a set of algorithms $\{\varphi_j(t)\}$, ($j = 1, 2, \ldots, 6$) and is realized by local servos, φ_j—servos positioning the ankle, knee, and hip joint of each leg (Fig. 18). The actuators are pneumatic servo motors. Electronic function generators deliver a set of signals $\{\varphi_j(t)\}$ which represent the correspondent joint position, in accordance with the set of algorithms. Signals $\{\varphi_j(t)\}$ are independently generated but are properly timed and synchronized [7].

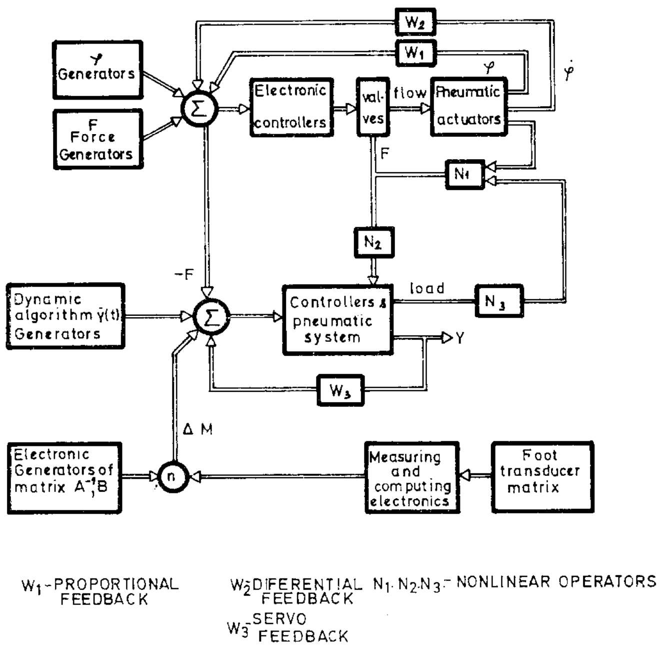

FIG. 18. Preliminary organization scheme of biped locomotion special purpose computer.

Regarding the nature of the system, it is obvious that φ_j—servos are heavily loaded and that the load changes very much as a function of time. More than that, the changes of load are very fast in some phases of gait (landing of foot). On the other hand, the delicate problem of biped stability excludes the "high-gain servo" approach, to overcome the stability problem.

As the biped locomotion system is moved, forced by the kinematic dynamic algorithm, it is possible to solve the servo loads $\bar{F}_{Lj}(t)$ analytically, assuming that the system obeys the algorithm. Transforming the $F_{Lj}(t)$ in the equivalent input signal $K_j(t)$ is accomplished and the result is superimposed upon the signal $\varphi_j(t)$ by a proper signal generator. This method permits the reasonable gain servo to overcome the fluctuations of $F_{Lj}(t)$

in the vicinity of the estimated $F_{Lj}(t)$. The difficulty in this method lies in the estimation of the nonlinear transfer function, which gives the equivalent input signal $K_j(t)$.

It should be mentioned that in this way, in principle the realization of the "internal" coordinates according to the kinematic-dynamic program of the locomotion system can be ensured. However, in order to ensure stable gait, it is necessary, as already said, to ensure the correction of the perturbed external coordinates. In order to achieve this, the compensating system must contain a supplementary feedback, representing some of the proposed compensations, linked to the fixed coordinates system.*

In the case where we decide for the compensating system via force measurement, it is indispensable to form a supplementary pressure feedback loop (Fig. 18). This compensating method requires pressure transducers. At this, the realization of relations 35 must be ensured, enabling calculation of the moment increments ΔM_x and ΔM_y, due to the redistribution of the vertical reaction F_z components. The mentioned supplementary compensating block, according to the proposed scheme (Fig. 18), also must ensure the realization of relations 25, 29, 30. The last relation gives the possibility of active moments ΔM corrections realization in the corresponding drives of the locomotion mechanism.

In this case the supposition still holds that such perturbations are in question, enabling the consideration of only the perturbed accelerations of the corresponding angular locomotion system coordinates. The block scheme in Fig. 18 contains such limitations, namely the supposition about the constant system phase coordinate vector

$$Y = \begin{Bmatrix} \vartheta \\ \psi \\ \dot{\vartheta} \\ \dot{\psi} \end{Bmatrix},$$

in the perturbed regime conditions.

This block scheme of the control system represents one of its possible realizations. It has to be emphasized once again that this scheme is in fact characterized by an independent feedback with respect to acceleration and thus, under the supposition of instant or approximately instant response of the actuators, the correction of the locomotion system inertiality is made possible, not "impairing" thereby the state coordinates.

Nevertheless, it should be pointed out that the regulating scheme demonstrated cannot be autonomous but should be integrated in a general scheme that would adjust the violated state coordinates of the Y-vector too [see Eqs. (23, 24)].

* In our case, that is the system, connected to the momentary foot contact with the support (or in the general case to the prescribed zero-moment point).

The authors express their gratitude to Vidojko Ćirić Ph.D. and Miroslav Zečević Dipl. Eng. for their elaboration of the computer algorithm, and useful comments in preparing the control schemes.

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