Impedance / Admittance Control

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Impedance

- **Mechanical impedance** is a measure of how much a structure resists motion when subjected to a given force. It relates forces with velocities acting on a mechanical system. The mechanical impedance of a point on a structure is the ratio of the force applied to the point to the resulting velocity at that point.
- Mechanical impedance is the inverse of mechanical admittance or mobility. The mechanical impedance is a function of the frequency ω of the applied force and can vary greatly over frequency. At resonance frequencies, the mechanical impedance will be lower, meaning less force is needed to cause a structure to move at a given velocity. The simplest example of this is when a child pushes another on a swing. For the greatest swing amplitude, the frequency of the pushes must be near the resonant frequency of the system.
- Where, is the force vector, is the velocity vector, is the impedance matrix and ω is the angular frequency.
- Mechanical impedance is the ratio of a potential (e.g. force) to a flow (e.g. velocity) where the arguments of the real (or imaginary) parts of both increase linearly with time. Examples of potentials are: force, sound pressure, voltage, temperature. Examples of flows are: velocity, volume velocity, current, heat flow. Impedance is the reciprocal of mobility. If the potential and flow quantities are measured at the same point then impedance is referred as driving point impedance; otherwise, transfer impedance.
- Resistance - the real part of an impedance.
- Reactance - the imaginary part of an impedance.
- http://en.wikipedia.org/wiki/Mechanical_impedance

Admittance

- In **mechanical systems** (particularly in the field of haptics), an admittance is a dynamic mapping from force to motion. In other words, an equation (or virtual environment) describing an admittance would have inputs of force and would have outputs such as position or velocity. So, an admittance device would sense the input force and "admit" a certain amount of motion.
- Similar to the electrical meanings of admittance and impedance, an impedance in the mechanical sense can be thought of as the "inverse" of admittance. That is, it is a dynamic mapping from motion to force. An impedance device would sense the input motion and "impede" the motion with some force.
- An example of these concepts is a virtual spring. The equation describing a spring is Hooke's Law,
- If the input to the virtual spring is the spring displacement, x, and the output is the force that the virtual spring applies, F, then the virtual spring would be classified as an impedance. If the input to the virtual spring is the force applied to the spring, F, and the output is the spring displacement, x, then the virtual spring would be classified as an admittance.
- http://en.wikipedia.org/wiki/Admittance

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Outline

- Introduction (1 slide)
  - Manipulators are not isolated systems
  - Interaction with environment
    - Environment either admits (admittance mode i.e. cartesian control) or impedes (impedance mode)
  - Impedance control
- Motivation (3 slides)
  - Manipulation scenario or two
  - What doesn't work (errors in joint angles, environmental unknowns)
  - What happens if you encounter these errors (movies here?)
- How impedance control works High Level (2 slides)
- Details on how ic works (2 slides)
- My implementation (3 slides)
  - Movie of it
- Alternate approaches (2 slides)
  - Force Control
  - Hybrid Control
- Summary and questions (1 slide)

Introduction

- Robotic manipulator control falls into one of two categories:
  - Unconstrained Control
    - The manipulator does not interact with its environment. Examples of this are robots that paint cars. Initially, all successful robotic applications were of this variety.
  - Force Control

Unconstrained Control

- Position (and velocity) of the end effector are controlled
- Useful for contact free activities like spray paining, and acceptable for trivial object manipulations (position control is good enough, and high degree of knowledge of manipulator and environment states allow for position control to work)
- Interactions with the environment are viewed as disturbances to the controller
- Controllers, trying to minimize position error increase force…often to the point that either the environment or the manipulator are damaged. Also leads to saturation and instability.

Constrained and Compliant Motion

- Control the dynamics of a manipulator in an environment.

Impedance Control

- Hogan showed that command and control of a vector such as position or force is not enough to control the dynamic interactions between a manipulator and its environment.
- The problem with hybrid position-force control is its failure to recognize the importance of manipulator impedance.
- Impedance control overcomes this problem, but ignores the distinction between position and force controlled subspaces, and no attempt is made to follow a commanded force trajectory.
- Hybrid Impedance Control (HIC) was introduced to overcome this shortcoming of IC.

- The main idea of impedance control is to make the behavior of each DOF of the manipulator to look like a mass-spring-dashpot system.
- Seen from the environment along any degree of freedom, physical systems come in only two types: admittances, which accept effort (force) inputs and yield flow (motion) outputs; and impedances, which accept flow (motion) inputs and yield effort (force) outputs.
- With a manipulator and environment, along a single DOF, one is always admittance while the other is impedance. I.e. When the environment is admittance (accepts force and yields flow), the manipulator is impedance (accepts motion and yields force).

- Impedance control guarantees that the arm approaches the environment with a velocity that can be properly limited in order to reduce impact forces.
- In practice when forces are exerted on the end effector, the desired trajectory to the manipulator is modified to reduce the forces.

Implementation 1

- The lowest order term in any impedance is the static relationship between output force and input displacement – or stiffness
- To implement impedance control, we assume actuators that are capable of generating commanded torques (T_{act}), joint angle sensors,
- $F_{int} - K[X_0 - X]$
- $dX = J(\theta)d\theta$
- $T_{act} = J'(\theta)F_{int}$
- $T_{act} = J'(\theta)K[X_0 - L(\theta)]$
- $F_{int} = B[V_0 - V]$
- $V = J(\theta)\omega$
- $T_{act} = J'(\theta)B[V_0 - J(\theta)\omega]$

Implementation II

- $M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$
- $F_{int} = F(X, V) - M\frac{dV}{dt}$
- $F_{int} = F(X_0 - X, V_0 - V) - M\frac{dV}{dt}$

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Implementation III

- $F_{int} = K[X_0 - X] + B[V_0 - V] - M\frac{dV}{dt}$
- $M_e\frac{dV}{dt} = F_{ext} + F_{int}$
- $(M_e + M)\frac{dV}{dt} = K[X_0 - X] + B[V_0 - V] + F_{ext}$
- $I(\theta)\frac{d\omega}{dt} + C(\theta, \omega) + V(\omega) + S(\theta) = T_{act} + T_{int}$

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Implementation IV

- $T_{act} = I(\theta)J^{-1}(\theta)M^{-1}K[X_0 - L(\theta)] + S(\theta) + I(\theta)J^{-1}(\theta)M^{-1}B[V_0 - J(\theta)\omega] + V(\omega) + I(\theta)J^{-1}(\theta)M^{-1}F_{int} - J'(\theta)F_{int} - I(\theta)J^{-1}(\theta)G(\theta,\omega) + C(\theta,\omega)$

Impedance Modulation Without Feedback

- $\omega = Y(\theta)h$
- $h = J'(\theta)p$
- $V = W(\theta)p$
- $W(\theta) = J(\theta)Y(\theta)J'(\theta)$
- $Ek = \frac{1}{2}h'Y(\theta)h$
- $p = M(\theta)V$

Impedance Modulation Using Actuator Redundancies

- $$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} K_s & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$
- $$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} K_s + K_t & 0 \\ 0 & K_e + K_t \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

Cartesian Control

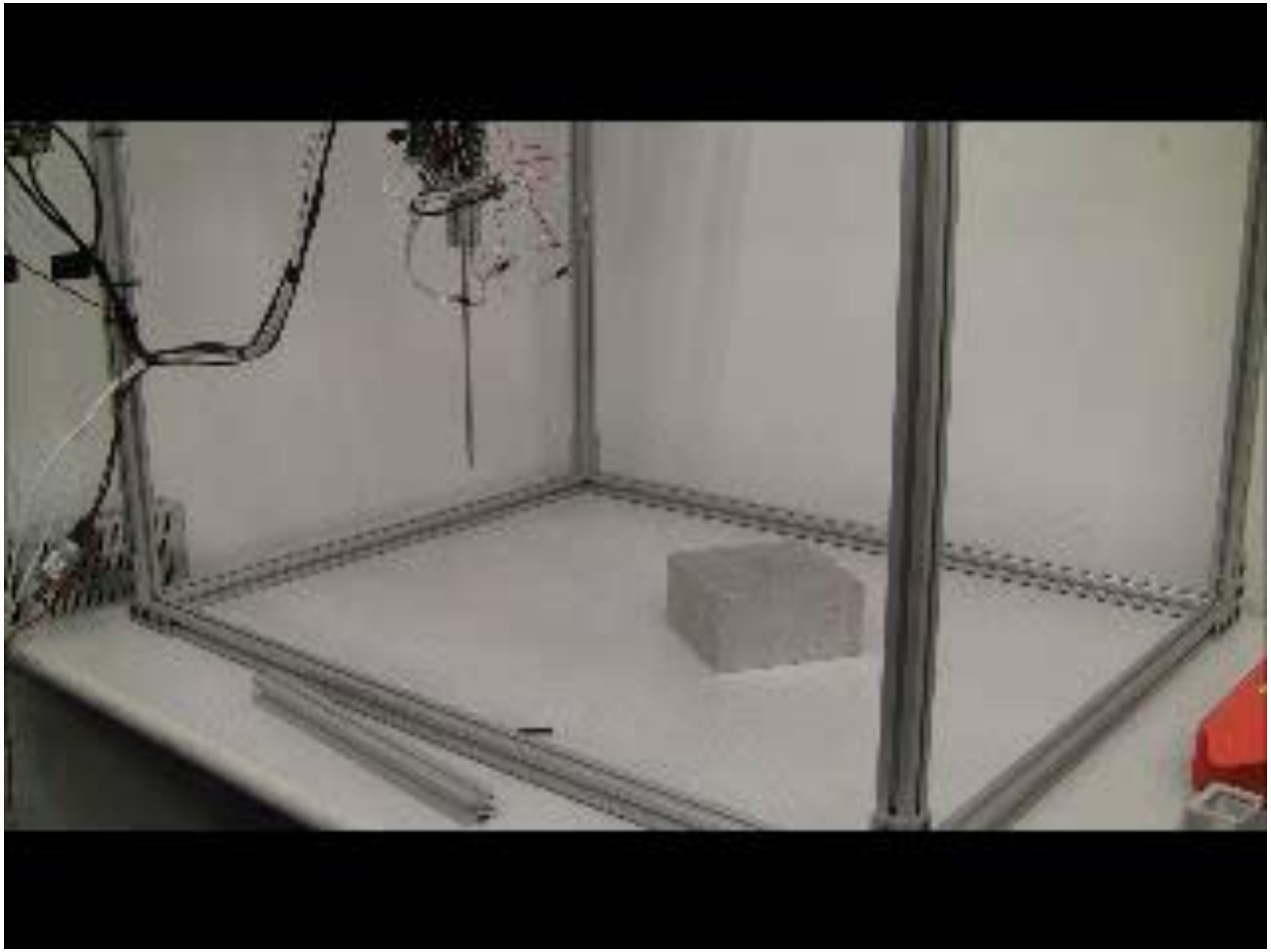
- We can control a manipulator to move through space without regard to environmental interactions using a PD controller:

$$\tau(XYZ) = K_p(XYZ_d' - XYZ) + K_d(X\dot{Y}Z_d' - X\dot{Y}Z)$$

Where:

- $\tau(XYZ)$ is torque (or more generally force) commanded to joints to provide XYZ movement
- K_p and K_d are proportional and derivative control coefficients
- XYZ_d' and $X\dot{Y}Z_d'$ are desired end effector position and velocity trajectories
- XYZ and $X\dot{Y}Z$ are current end effector position and velocity

Cartesian Control with Obstacle



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Force Feedback

- Assuming we have force sensors mounted to the end effector of our manipulator, we can sense interaction forces in the end effector's local coordinate system *(Cf_x, Cf_y, Cf_z)*
- Use knowledge of the manipulator's configuration to create a rotation matrix to convert the forces to the manipulator's base coordinate system *(f_x, f_y, f_z)*

$$f^* = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \begin{bmatrix} Cf_x \\ Cf_y \\ Cf_z \end{bmatrix}$$

Calculate Trajectories

- Based on force feedback, calculate desired position (XYZ'_d) and velocity ($X\dot{Y}Z'_d$) trajectories

$$XYZ'_d = XYZ_d - \mathcal{L}^{-1}\left\{\frac{f^*}{As^2 + Bs^2 + K}\right\}$$

$$X\dot{Y}Z'_d = X\dot{Y}Z_d - \mathcal{L}^{-1}\left\{\frac{sf^*}{As^2 + Bs^2 + K}\right\}$$

Where A, B and K are impedance control filter parameters and s is the Laplacian variable

Impedance Control

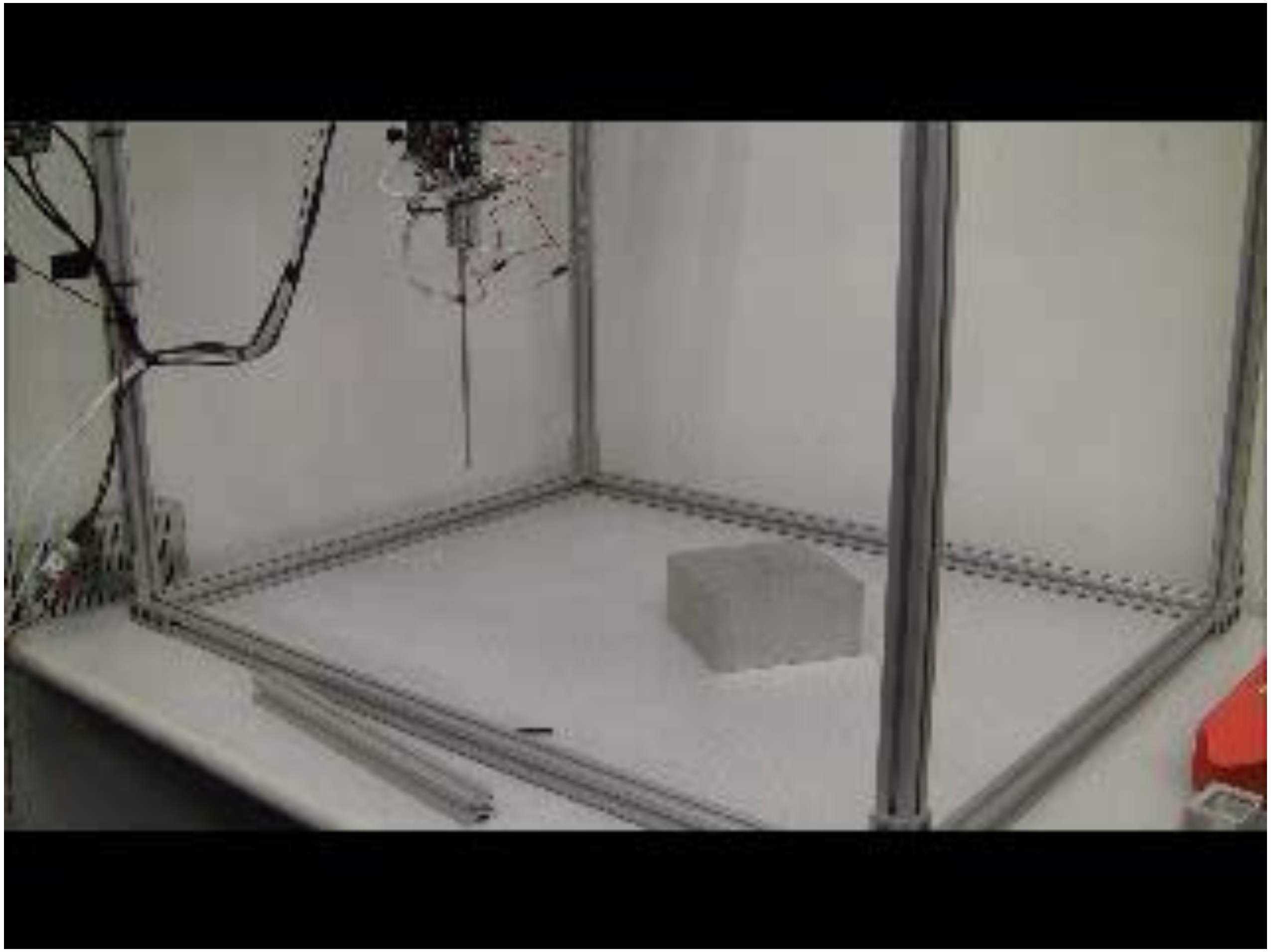
- Using our force feedback adjusted trajectories as goals, perform PD Cartesian control

$$\tau(XYZ) = K_p(XYZ'_d - XYZ) + K_d(X\dot{Y}Z'_d - X\dot{Y}Z)$$

- When specifying actual torques to send to individual joints, we also need to consider the manipulator's inertia (M), gravity (G), Coriolis and centripetal torque (C) and viscous and Colomb friction (F). These are added to the transpose of the Jacobean which represents ???

$$\tau(q) = J^T\tau(XYZ) + M(q)\ddot{q} + C(q,\dot{q})\ddot{q} + F(\dot{q}) + G(q)$$

Impedance Control with Obstacle



Impedance Control, Hand Interaction