Compliance Control Basic

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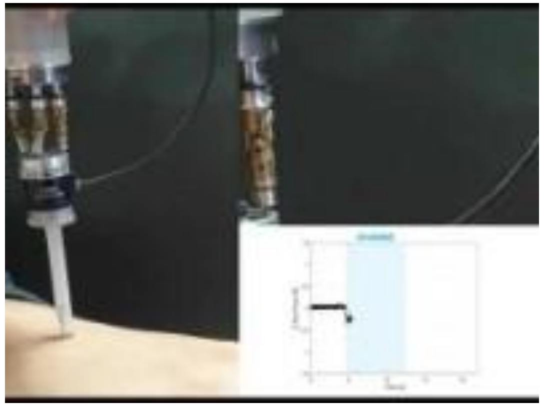
Force Control? Compliance Control? Impedance/Admittance Control?

Force Control : Control method to modify trajectory by receiving feedback of force and position values

- Can be implemented when there is force and position feedback in contact with an object
- The position can be calculated by joint angle, for force torque sensor or joint torque sensor or current sensor.



1.Contact Force Control

- If the force in contact is important
  - After contact, moving while maintaining a constant force (for example, grinding)

2.Compliance Control

- When contact requires flexible movement
  - If the task must be executed smoothly and in motion during contact (for example, precision insertion such as gear engagement)

Force Control? Compliance Control? Impedance/Admittance Control?

Impedance/Admittance Control : Implements impedance, a correlation between exercise and external force

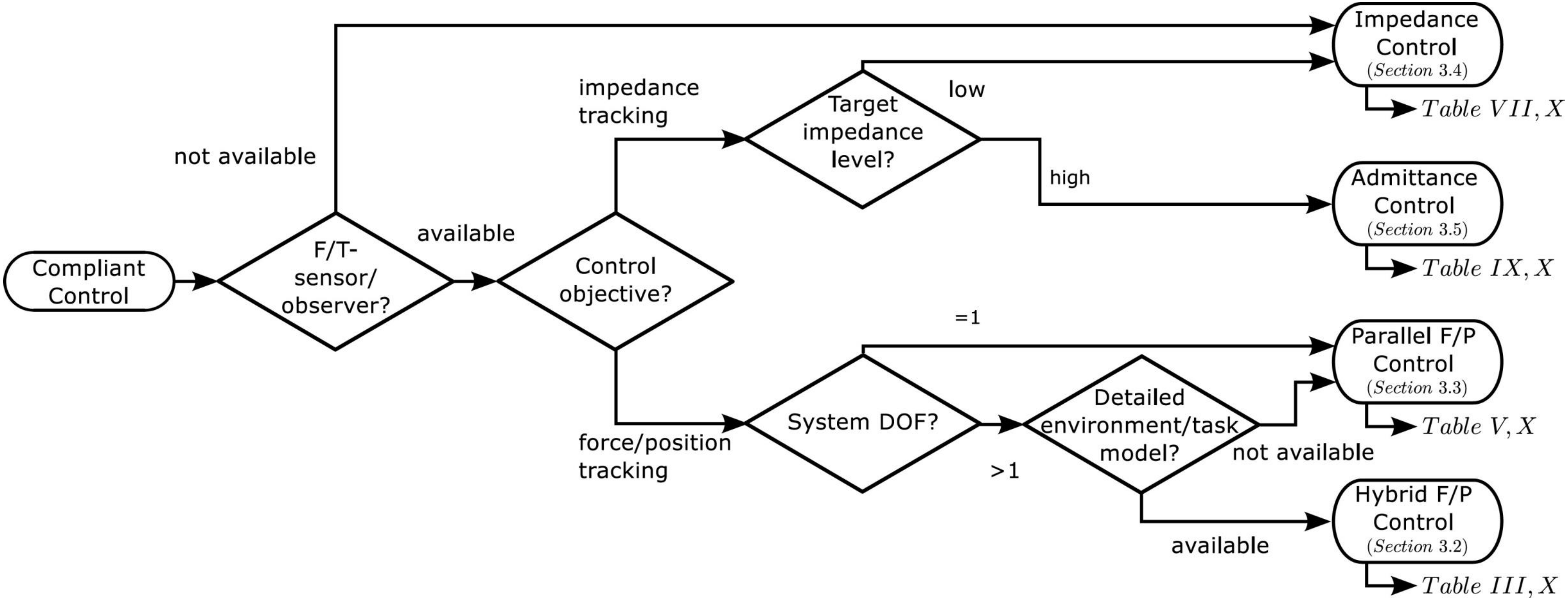
1.Impedance Control

- Force/Torque based
- Control of the final output of the force using the input value, speed
- Advantages: High response speed based on torque control
- Disadvantage: Large position error even with small modeling error (effect of Cascade control of motor control)

2.Admittance Control

- Position based
- Control of the final speed output using the force that is the input value
- Advantages: Accurate position control is possible by robustness at modeling errors and disturbances
- Disadvantages: Slower response than Force based impedance control (effect of Cascade control of motor control), abnormal impedance behavior even with slight position error

When using Impedance/Admittance Control?



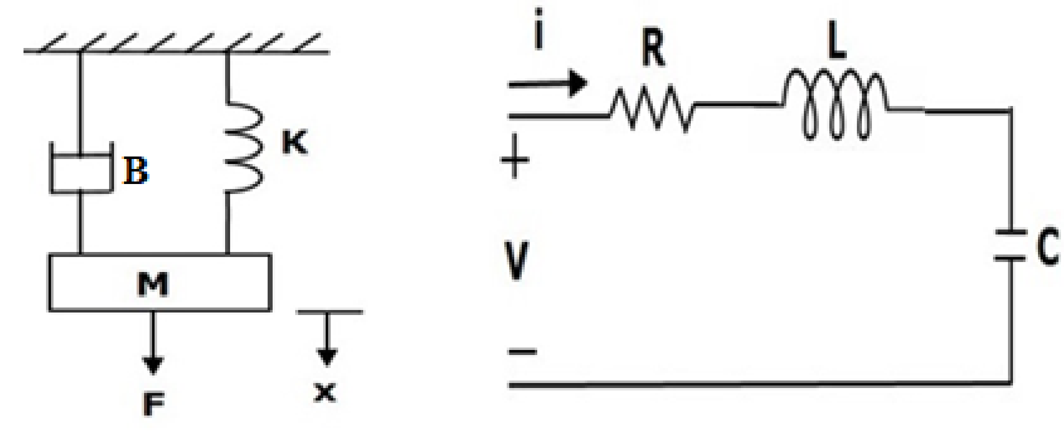
What is Impedance?

Impedance (Z) : The opposition to the flow of alternating current in a circuit.

$$V = Z * I$$

How about mechanical Impedance?

Mechanical system	Electrical system
Force (F)	Voltage (V)
Mass (M)	Inductance (L)
Frictional Coefficient (B)	Resistance (R)
Spring Constant (K)	Reciprocal of Capacitance (1/C)
Displacement (x)	Charge (q)
Velocity (v)	Current (i)



Mechanical System

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Electrical System

$$V = Ri + L\frac{di}{dt} + \frac{1}{c}\int i dt$$

Impedance Control

- Force/Torque based
- Control of the final output of the force using the input value, speed

1. Impedance Control with Current Sensor(without F/T Sensor)

- Estimate External Force with Current Sensor
- Do not need External F/T Sensor -> Hard to implement

2. Impedance Control with F/T Sensor

- Measure External Force with F/T Sensor
- Need to use External F/T Sensor -> Expensive
- Easy to implement

Impedance Control

1. Impedance Control with Current Sensor(without F/T Sensor)

- Estimate External Force with Current Sensor
- Do not need External F/T Sensor -> Hard to implement

$$\mathrm{M_d}(\ddot{x}_d - \ddot{x}) + \mathrm{D_d}(\dot{\mathrm{x}}_\mathrm{d} - \dot{x}) + \mathrm{K_d}(\mathrm{x_d} - \mathrm{x}) = F$$

$u = \underline{M(q)}\ddot{q} + \underline{n(q,\dot{q})} + \underline{J^T(q)F_e}$ -> Output Joint Dynamics

$J^T(q)F_e$ External Force, Jacobian => change task space to joint space

$n(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q) + \tau_{fr}(\dot{q})$ Coriolis Force, gravity, friction etc..

$M(q) = J_l(q) + J_m$ (rigid body Motor(Actuator) Inertia)

$u_c = \underline{M(q)}y + \underline{\hat{n}(q,\dot{q})}$ -> Input Joint Dynamics

$\hat{n}(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q) + \tau_{fr}(\dot{q})$ Coriolis Force, gravity, friction etc..

$M(q) = J_l(q) + J_m$ (rigid body Motor(Actuator) Inertia)

Impedance Control

1. Impedance Control with Current Sensor(without F/T Sensor)

$$u_c = u$$

$$M(q)y + \hat{n}(q,\dot{q}) = M(q)\ddot{q} + n(q,\dot{q}) + J^T(q)F_e$$

$$\Delta\ddot{q} = y - \ddot{q} = M^{-1}(q)J^T(F_e + J^{-1}\Delta n(q,\dot{q}))$$

$$F = ma \Rightarrow M_d J \Delta\ddot{q}$$

$$= M_d J M^{-1}(q)J^T(F_e + J^{-1}\Delta n(q,\dot{q}))$$

$$M_d(\ddot{x_d} - \ddot{x}) + D_d(\dot{x_d} - \dot{x}) + K_d(x_d - x) = F$$

$$M_d(\ddot{x_d} - \ddot{x}) + D_d(\dot{x_d} - \dot{x}) + K_d(x_d - x) = M_d \boxed{J M^{-1}(q)J^T}(F_e + J^{-1}\Delta n(q,\dot{q}))$$

Make system coupled
=> Hard to implement desired impedance

Coupled system?
⇒ System is been Multi input Multi output system

Impedance Control

2. Impedance Control with F/T Sensor

- Estimate External Force with Current Sensor
- Do not need External F/T Sensor -> Hard to implement

$$\mathrm{M_d}(\ddot{x}_d - \ddot{x}) + \mathrm{D_d}(\dot{\mathrm{x}}_\mathrm{d} - \dot{x}) + \mathrm{K_d}(\mathrm{x_d} - \mathrm{x}) = F$$

$u = \underline{M(q)}\ddot{q} + \underline{n(q,\dot{q})} + \underline{J^T(q)F_e}$ -> Output Joint Dynamics

$J^T(q)F_e$ External Force, Jacobian => change task space to joint space

$n(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q) + \tau_{fr}(\dot{q})$ Coriolis Force, gravity, friction etc..

$M(q) = J_l(q) + J_m$ (rigid body Motor(Actuator) Inertia)

$u_c = \underline{M(q)}y + \underline{\hat{n}(q,\dot{q})} + \underline{J^T(q)F_s}$-> Input Joint Dynamics

$J^T(q)F_e$ Sensing Force, Jacobian => change task space to joint space

$\hat{n}(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q) + \tau_{fr}(\dot{q})$ Coriolis Force, gravity, friction etc..

$M(q) = J_l(q) + J_m$ (rigid body Motor(Actuator) Inertia)

Impedance Control

2. Impedance Control with F/T Sensor

$$u_c = u$$

$$M(q)y + \hat{n}(q,\dot{q}) + J^T(q)F_s = M(q)\ddot{q} + n(q,\dot{q}) + J^T(q)F_e \qquad When, F_s = F_e$$

$$\Delta\ddot{q} = y - \ddot{q} = M^{-1}(q)\Delta n(q,\dot{q})$$

$$F = ma \Rightarrow M_d J \Delta\ddot{q}$$

$$M_d(\ddot{x}_d - \ddot{x}) + D_d(\dot{x}_d - \dot{x}) + K_d(x_d - x) = F_s + M_d J M^{-1}(q)\,(\Delta n(q,\dot{q}))$$

$\Rightarrow$ Uncoupled system
$\Rightarrow$ Can implement desired impedance

Admittance Control

1. Admittance Control with F/T Sensor

$$f_{ext} - J\tau_{des} = M\Delta\ddot{x}_a + C\Delta\dot{x}_a + K\Delta x_a$$

$$\Delta\ddot{x}_a = M^{-1}((f_{ext} - J\tau_d) - C\Delta\dot{x}_a - K\Delta x_a)$$

$$J\Delta\ddot{q} + \dot{J}\Delta\dot{q} = M^{-1}((f_{ext} - J\tau_d) - CJ\Delta\dot{q} - KJ\Delta q)$$

$$\ddot{q} = J^{-1}M^{-1}((f_{ext} - J\tau_d) - (M\dot{J} + CJ)\Delta\dot{q} - KJ\Delta q)$$

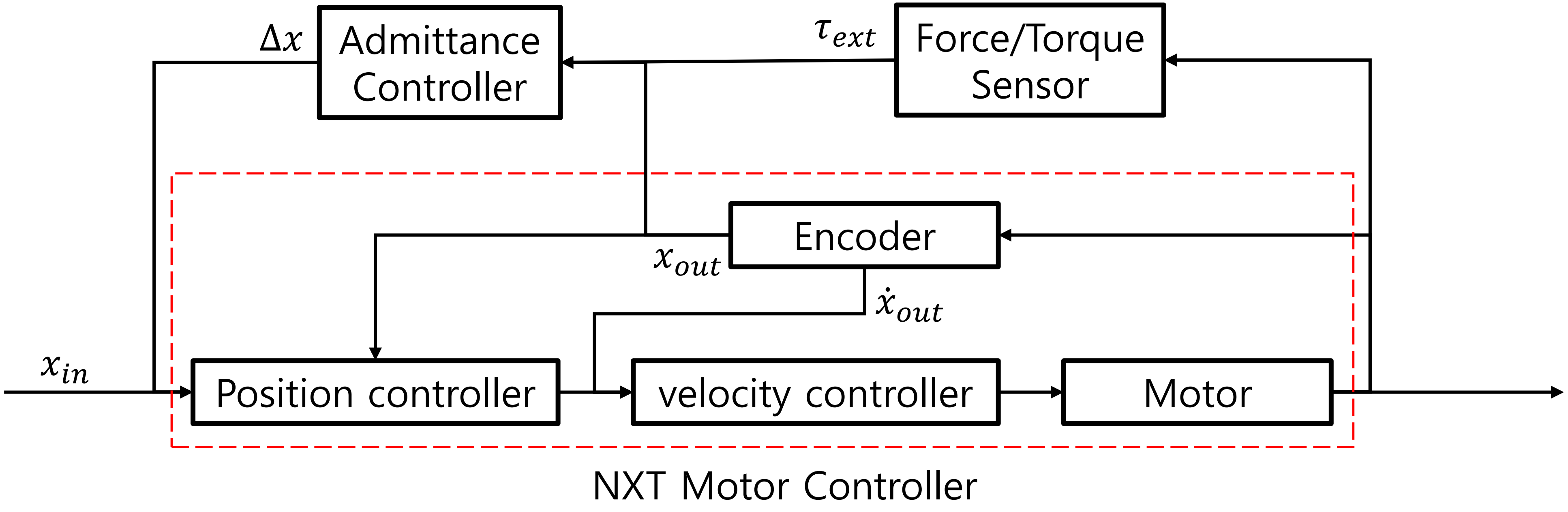
$$\dot{q} = \dot{q}_0 + \ddot{q}\Delta t$$
$$q = q_0 + \dot{q}\Delta t$$

$$\Delta\ddot{x} = J\Delta\ddot{q} + \dot{J}\Delta\dot{q}$$
$$\Delta\dot{x} = J\Delta\dot{q}$$

Compliance Control with NXT

- NXT Motor cannot torque/current based control
- Implement Admittance control with NXT

Admittance Control diagram



Compliance Control with NXT

Simple Admittance Control(1-DOF)

$$\ddot{q} = J^{-1}M^{-1}((f_{ext} - J\tau_d) - (M\dot{J} + CJ)\Delta\dot{q} - KJ\Delta q)$$

$$\begin{aligned}
M &= Mass(Desired\ System\ Mass, Constant)\\
C &= Damper(Desired\ System\ Damper, Constant)\\
K &= Spring(Desired\ System\ Spring\ Constant)\\
J &= r\ (1 - DOF\ system, link\ length)\\
\tau_d &= 0\ (Constant)
\end{aligned}$$

$$=>\ \ddot{q} = \frac{1}{r}\frac{1}{M}((f_{ext}) - (C * r)\Delta\dot{q} - K * r\Delta q)$$

$$\begin{aligned}
\dot{q} &= \dot{q}_0 + \ddot{q}\Delta t\\
q &= q_0 + \dot{q}\Delta t
\end{aligned}$$