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Hands-on Lab:

LabVIEW – System Identification

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The LEGO Damped Compound Pendulum (DCP) is an example of a second order system. Using the previous Angle Sensor lab, one can measure the DCP's free fall response. As a pendulum, the oscillations will exponentially decrease. One can then calculate the damping ratio and damped natural frequency. This results in system identification, namely to determine the DCP's characteristic equation and consequently, the transfer function.

Concept 1: DCP Step Response Testing

Step 1: Create Front Panel and Initial Block Diagram

From your Angle Sensor lab open the file `nxtLabviewEncoderGraphTest1_2.vi`. Recall that you wrote this file in Exercise 2-2. This Labview program consisted of a waveform chart, an XY chart, and wrote angle data every 25 milliseconds to a CSV file.

Save this file as `nxtLabviewLegoDcpSystemId1_0.vi`. Note **Errata 1 and Errata 2** in **Figure 1-1A right**.

Add a `toggle switch` (`MINDSTORMS Robotics – Boolean – Vertical Toggle Switch`) to the Front Panel and add text to this switch (**Figure 1A left**). Right click this switch to make sure that the `Mechanical Action` is set to `Switch When Pressed`. In the Block Diagram, drag the `toggle switch` so that it is inside the `while-loop` (**Figure 1A right**).
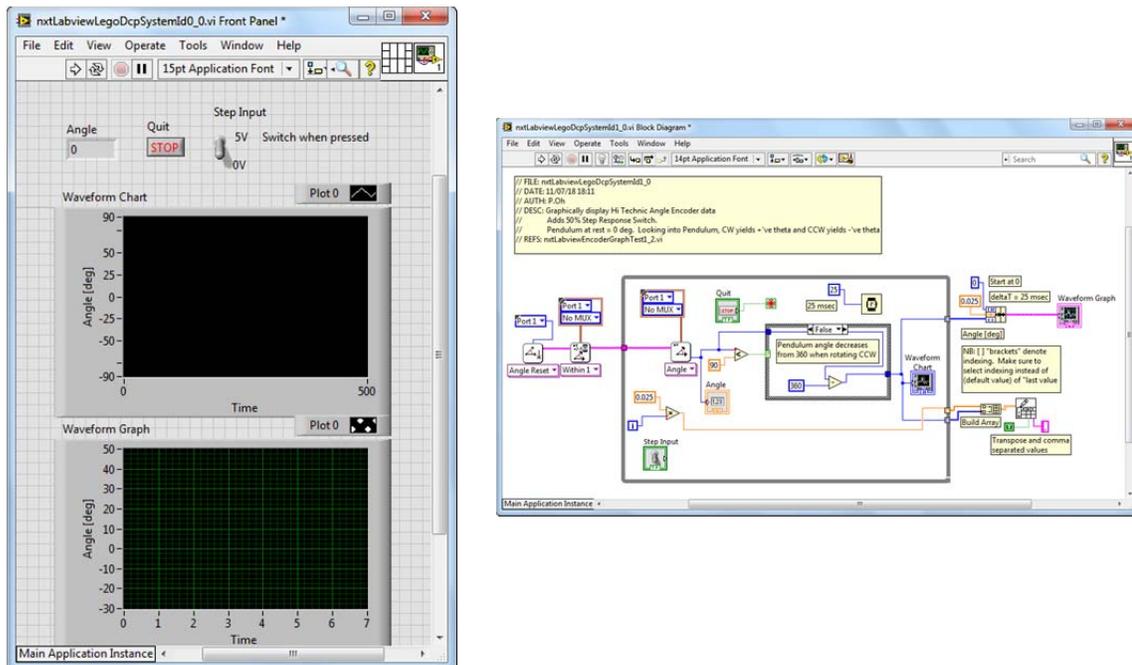
Figure 1-1A: The toggle switch is in the Front Panel (left) and inside while-loop of Block Diagram (right). **Errata 1:** The block diagram incorrectly compares the angle to 90. **This should be 180**. Also, **Errata 2**: the numeric indicator should be re-wired to be after the Case Structure (e.g. next to the waveform chart). Refer to **Figures 2B and 2C**, which are correct

Step 2: Add a Motor Block to the Block Diagram

The toggle switch serves as a step input to the motor; when flipped on, a 50% motor power command will be fed into the motor. To achieve this, first add the NXT motor block (`MINDSTORMS Robotics – I/O – Move Motors`) into the `while-loop`. By default LEGO Mindstorms calls this block `Power`. Right click on the `Power` block and select `Create Constant`. Hovering over the created constant, select `NXT Motor 1` (**Figure 1-2A**).
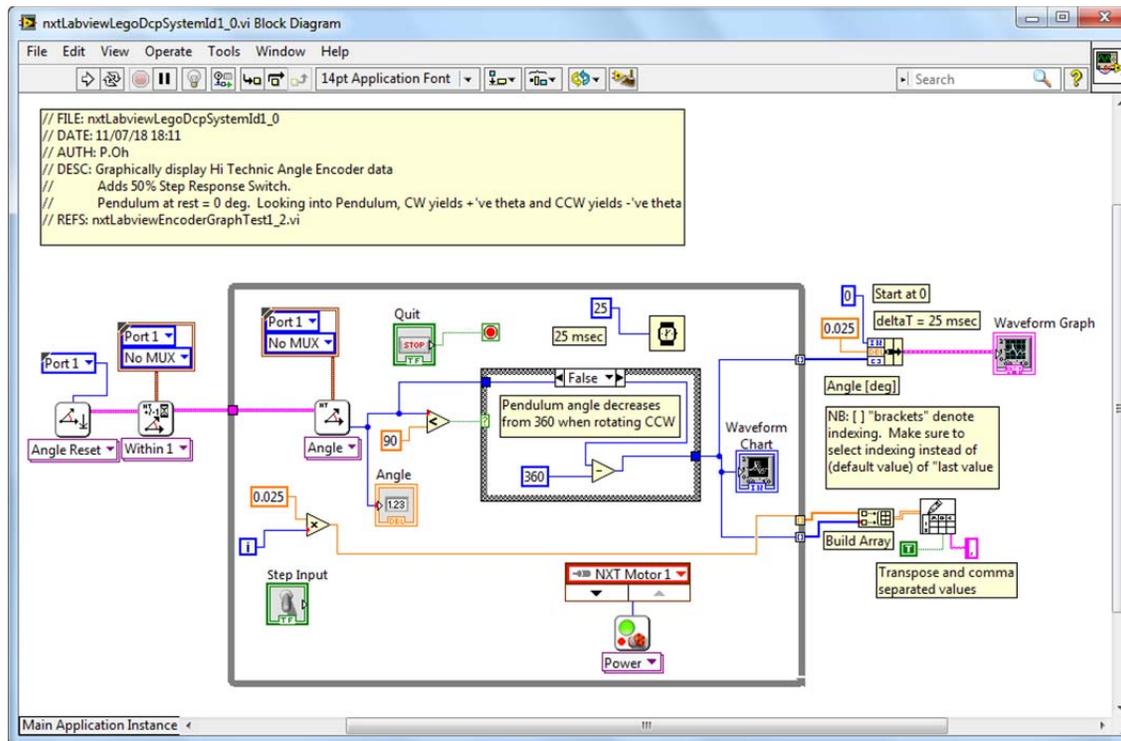
Figure 1-2A: Motor added to block diagram (called `Power`) with constant `NXT Motor 1` as the assigned port. NB: The figure is incorrect, recall **Errata 1** and **Errata 2** from Figure 1A. Refer to **Figures 2B and 2C**, which are correct

Next, place a `Case Structure` between the toggle switch and motor. For the `True` condition, add a `numeric constant`. Set this constant to 50, and wire it thru the `Case Structure` and into the motor's `Power` input (**Figure 1-2B**).

For the `False` condition, add a `numeric constant` with the value of `0`. Wire this constant to the `Case Structure` (**Figure 1-2C**). The net effect is when the Labview program is played toggling the switch will set the `Case Structure` to `True` and consequently issue a value of `50` into the motor power (and thus activating the motor). Conversely, when the switch is `False`, `0` is fed into the motor power (essentially deactivating the motor).

Save your program (called `nxtLabviewLegoDcpSystemId1_0.vi`).
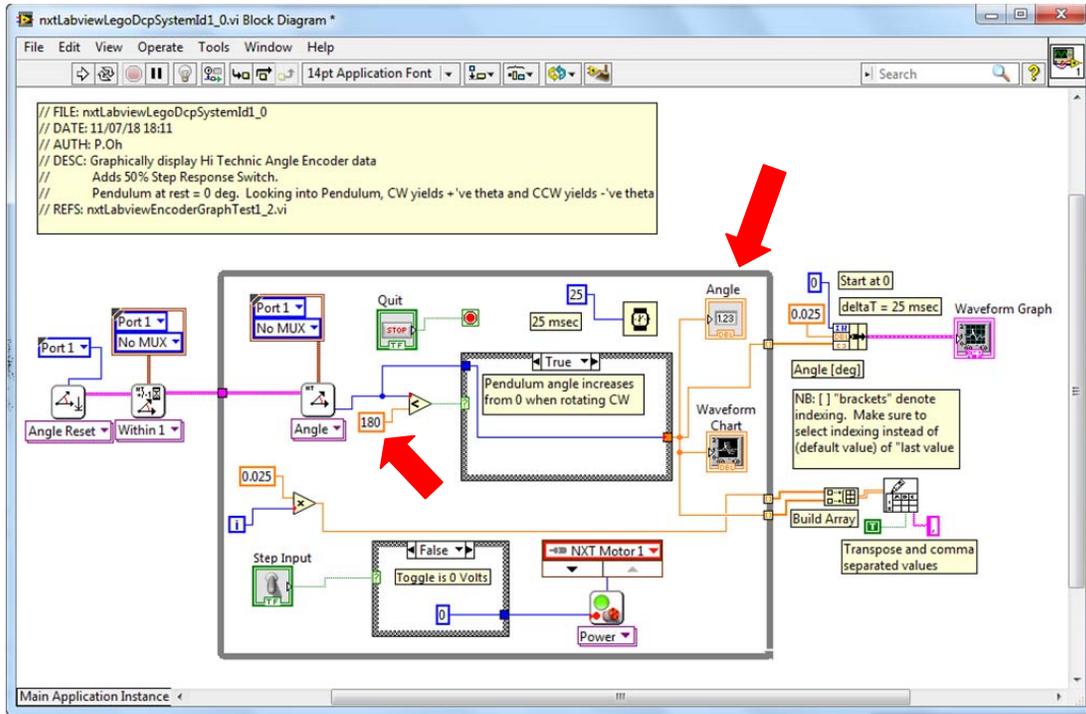
Figure 1-2B: A `Case Structure` is added into the Block Diagram. For the `True` condition, the `numeric constant` is fed into the input of the motor (`Power` block). The `toggle switch` is wired to the `Case Structure`. NB: NB: **The figure here is correct**; it incorporated the aforementioned errata
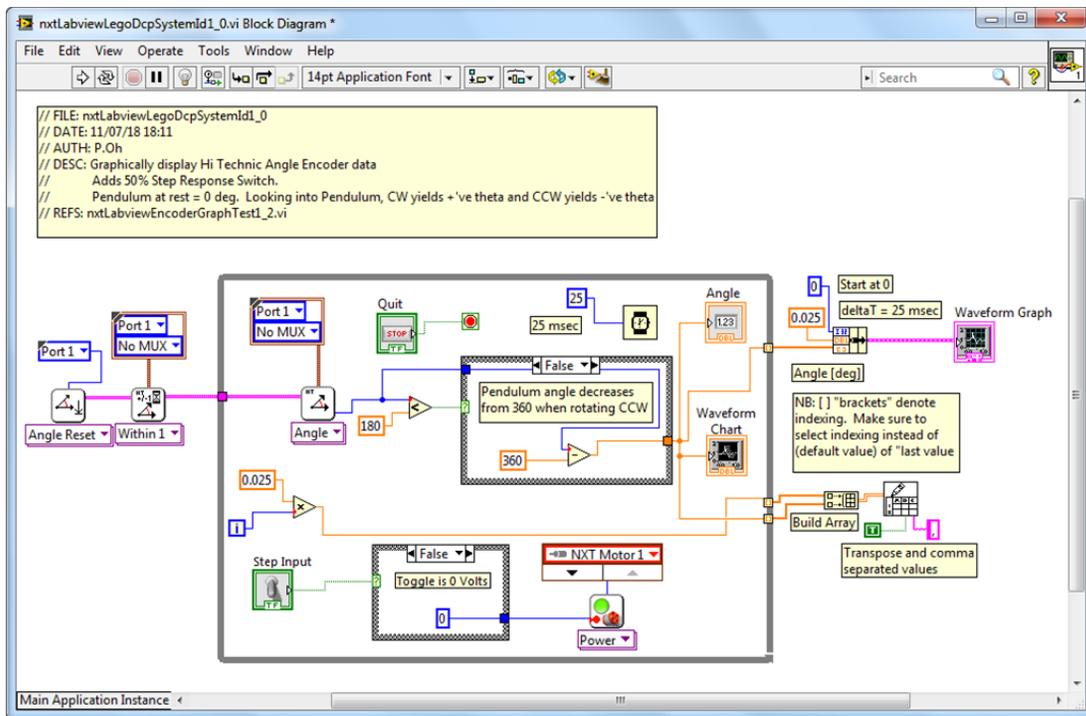
Figure 1-2C: For the Case Structure's `False` condition, a `numeric constant` of zero is fed into the motor input (`Power` block). NB: **The figure here is correct**; it incorporated the aforementioned errata

Step 3: Execution

A step response will be performed on the LEGO DCP (see **Figure 1-3A**). The above block diagram shows the angle sensor and motor connections. Thus, connect your NXT cable from the Hi Technic Sensor to the Port 1 of the NXT and your spliced NXT cable to Port A of the NXT.

Since this program will activate the motor-prop, make sure to clear any obstacles the pendulum could possibly hit. Execute the program (`nxtLabviewLegoDcpSystemId1_0.vi`) and observe the waveform chart. Toggling on the switch will send 50% power to the motor and the pendulum will start to rotate clockwise. Toggling off the switch stops the motor. Repeat toggling on and off to ensure the LEGO DCP and your program is working as expected (**Figure 1-3B**).

Hit the front panel's `Stop` button. A pop up box will prompt you to enter a file name. For example, saving as **systemId-110918-07H41M.csv** suggests the data was saved on 11/09/18 and 7:41 AM as comma-separated values (CSV) file format.

Opening this CSV file with Excel and plotting the values (**Figure 1-3C**) yields what was captured and displayed on the front panel's XY chart
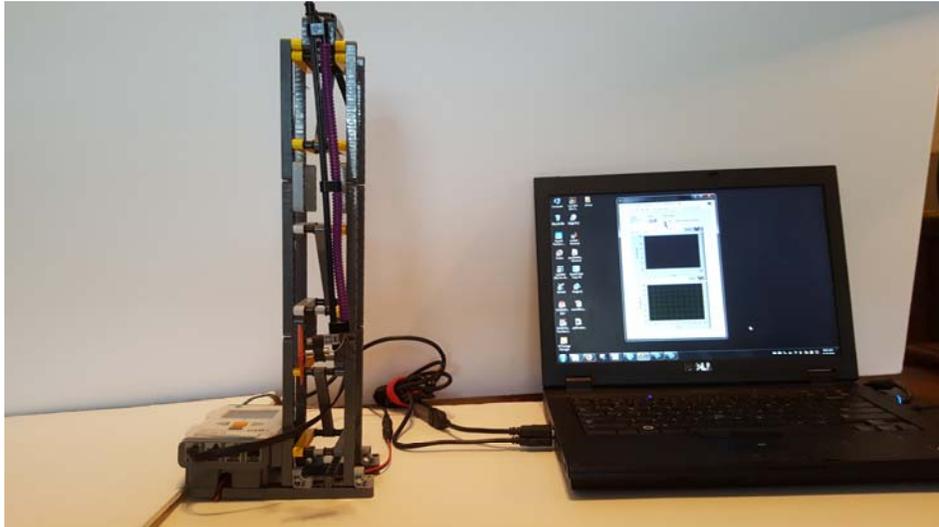
Figure 1-3A: LEGO DCP connected to laptop. Note: no obstacles in the pendulum's pathway. Angle sensor and Motor are connected to the Brick's Port 1 and Port A respectively.
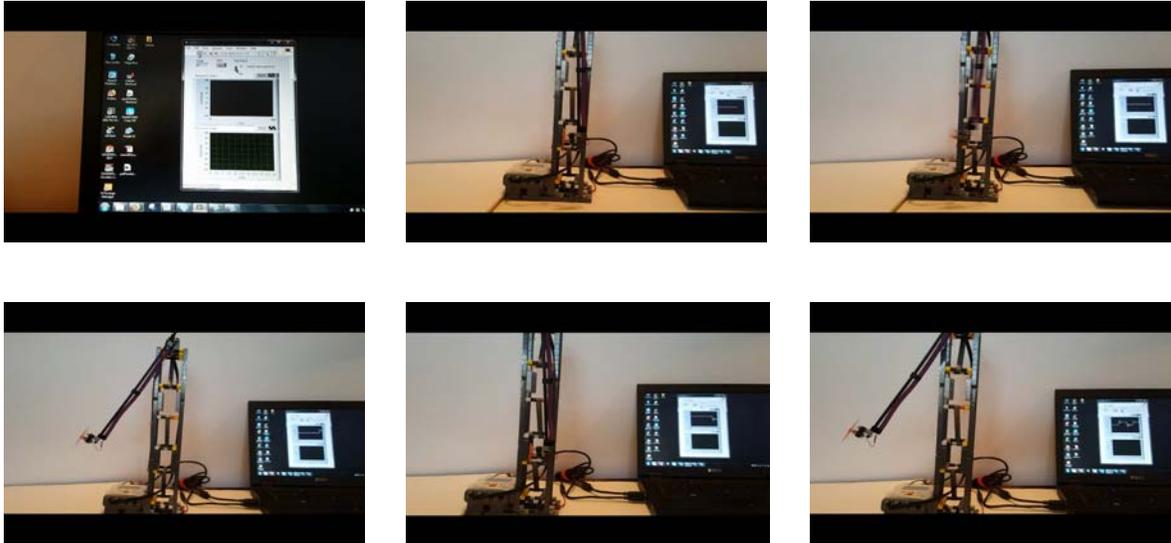
Figure 1-3B: Sequential photos from a video. Top Row: Program is executed (left); pendulum at rest (middle); and switch toggled on and activates motor-prop (right). Bottom Row: motor-prop generates thrust which raises pendulum and waveform chart provides real-time display of angle (left); toggling motor off, stops motor and pendulum returns to zero angle position (middle); toggling switch on again raises pendulum and waveform chart shows angle response (right).
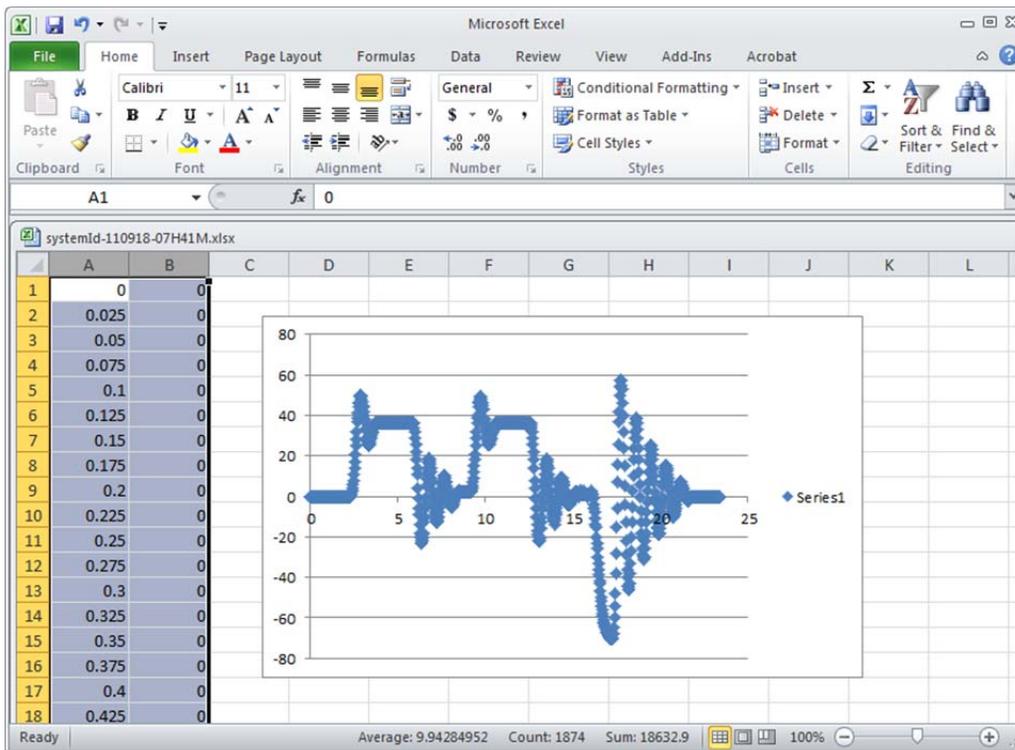
Figure 1-3C: Opening the CSV file in Excel, and performing a scatter plot, shows the LEGO DCP's angular response to a step input. One sees (around 2.5 seconds) that apply 50% motor power (due to toggling the switch on), has the pendulum reach a peak angle (about 50 degrees) and oscillates to reach a steady-state angle of about 31 degrees.

Exercise 1:

1.1 Complete the following table

Your Step Response Input (i.e. motor power) [%]		Your Steady-State Angle θ_{ss} [deg]	
e.g. 50%		e.g. 31 deg	

Concept 2: Free fall data capture and Logarithmic Decrement

Pendulums, like the LEGO DCP, when released from an initial angle, will oscillate. The oscillations' amplitudes become smaller as the pendulum swings back-and-forth, until the finally stop. This is a characteristic of damped second-order systems like the LEGO DCP. The amplitudes and period are related to the system's damping constant and damped natural frequency. This relationship will be shown in this concept. Such system identification serves to find a mathematical model (called transfer function) of the system (i.e. plant).

Step 1: Execute `nxtLabviewLegoDcpSystemId1_0.vi`

Here, toggling will not be used. Instead, run the program and rotate the pendulum counter clockwise (CCW) to an initial angle (e.g. -45 degrees) and release (**Figure 2-1A left**). Once the pendulum is motionless, hit the `Stop` button and save the data e.g. `legoDcpFreefall-111018-08H56M.csv`. Graph the data in Excel as a scatter plot (**Figure 2-1A right**).
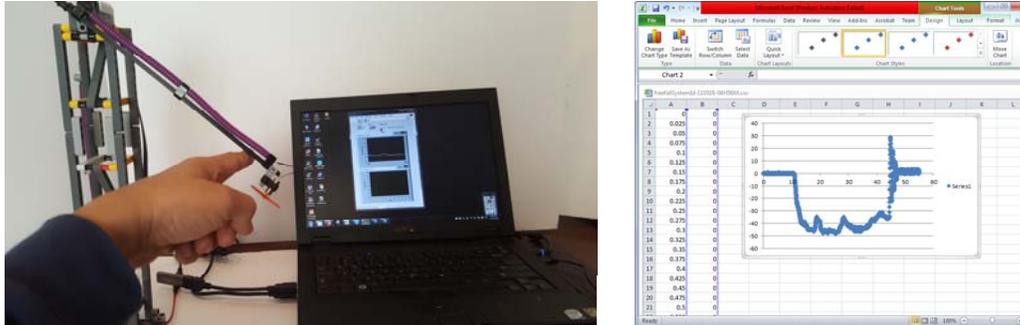
Figure 2-1A: Program is executed on laptop and pendulum moved counter clockwise to about -45 degrees (left). Scatter plot of captured data in Excel (right).

Step 2: Label data points for peaks and calculate damping ratio and natural frequency

The data of interest in **Figure 2-1A right**, is the oscillatory peaks. Recall that the pendulum's angle gets smaller as time elapses. The amplitudes of subsequent peaks are related to the pendulum's damping ratio ζ and natural frequency ω_n thru a logarithmic decrement (1):

$$\Delta \stackrel{\text{def}}{=} \ln\frac{a}{b} = \frac{\zeta 2\pi}{\sqrt{1 - \zeta^2}} \tag{1}$$

Here, a is the nth peak and b is the $(n + 1)$th peak (for any n) are the amplitudes of subsequent peaks. The period T of the oscillations is also related thru (2)

$$\frac{2\pi}{T} = \omega_n\sqrt{1 - \zeta^2} \tag{2}$$

Using (1) and (2) and with the definition of Δ one can show (3)

$$\zeta = \frac{\Delta}{\sqrt{\Delta^2 + 4\pi^2}} \qquad \text{and} \qquad \omega_n = \frac{2\pi}{T\sqrt{1 - \zeta^2}} \tag{3}$$

Figure 2-2A left focuses the scatter plot on the oscillatory peaks. Point your mouse over each peak to reveal the associated angle and time. For clarity, these angles and times are added as text boxes over each peak. The plot's title and axes are also labeled. **Figure 2-2A right** implements equations (1)-(3) in Excel. Here, the average period and logarithmic decrement were used.

time stamp	angle	deltaT	ln(a/b)
5.65	39		0.20
6.5	32	0.85	0.21
7.4	26	0.9	0.21
8.225	21	0.825	
	Average	0.858	0.21

zeta	omega [rad/s]	L [m]	d [m]	m_L [kg]
0.033	7.320	0.265	0.1325	0.0137

J [kg*m^2]	c [Nms/rad]
0.00033	0.00016
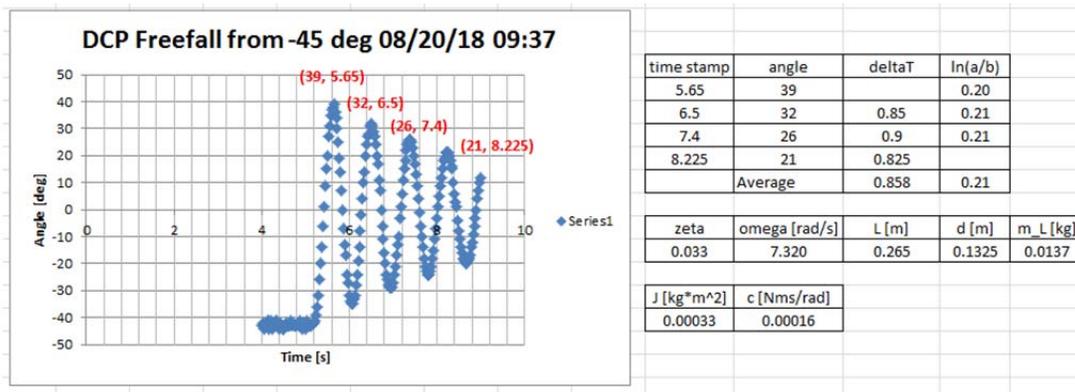
Figure 2-2A: From an initial angle of about -45 degrees, the pendulum oscillates for about 4 times. The angle and time of each peak was identified. The peaks are labeled with this data (left). The calculated logarithmic decrement, damping ratio, and coefficient of friction from equations (1)-(3) were calculated in Excel (right)

Congratulations: LEGO Damped Compound Pendulum System ID Completed

Exercise 2:

2.1. Complete the following table

Your Damping Ratio ζ		Your Natural Frequency ω_n	
e.g. 0.033		e.g. 7.320	

2.2. Referring to the free body diagram, complete the following table
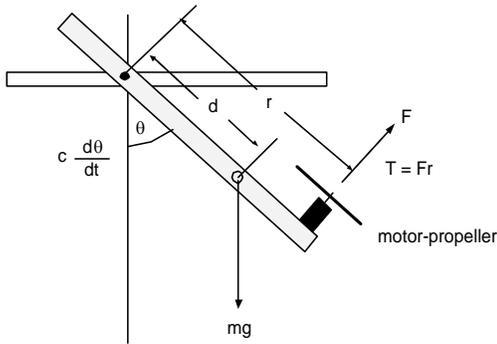
r	Pendulum Length [m]	e.g. 0.265 [m]	Your value
d	Pivot to CG distance [m]	e.g. 0.1325 [m]	
m_L	Mass of Pendulum [kg]	e.g. 0.0137 [kg]	

2.3. Using $J = \frac{m_L g d}{\omega_n^2}$ compute the Moment of Inertia (e.g. 0.00033 kgm^2)

2.4. Using $c = 2\zeta\omega_n J$ compute the coefficient of friction (e.g. 0.00016 Nms/rad)