**Homework – Block Diagrams with Units and State Space Realization**

1. Below is an antenna positioning system. Here, consider the control system for the yaw angle. Suppose that the power amplifier gain is and that the gear ratio and the angle sensor (the shaft encoder and data hold) are such that. The units for is volts and is degrees. Let be the input voltage to the motor. The transfer function of the motor pedestal is given by.





1. With the system open loop , a unit step function of voltage is applied to the motor []. Show that the steady-state output angle is degrees. Show that the steady-state output angular velocity is **(5-points)**
2. Using values stated in the problem, show that the transfer function with the gains and plant is the figure below **(5-points)**



1. Show that the block diagram changes to the one below when is in radians **(5-points)**



1. Given a step input of is applied at show that the response is **(5-points)**
2. Show that the response in (D) reaches steady-state in approximately 2 seconds **(5-points)**
3. Let the state variable model of a servomotor, given below, be equal to the transfer function of the motor pedestal (from Problem 1 above):

It can be shown that the solution for the above is. Also, from the block diagram of Problem 1-B, one has. Let and. Show that the state space realization is given as below **(5-points)**

1. Referring to Problem 1
2. Verify that the transfer function of Problem 1-B is given by , which yields the angle in degrees **(5-points)**
3. Show that the transfer function of 3-A such that is in radians is given by **(5-points)**
4. Verify the results of 3-B by showing the transfer function of 1-C is given by **(5-points)**
5. The figures below depict a satellite (left) and associated block diagram of the satellite’s attitude control system (right). Torque from the thrusters change the satellite’s yaw angle.





Since there is essentially no friction in space and assuming the satellite is rigid, then the moment of inertia about the yaw axis is related to the torque:

The Laplace transform yields:

Alternatively we have the plant transfer function:

1. Show that the transfer function **(5-points)**
2. Let and. Show that the closed-loop system’s state space realization is given by the equations below **(5-points)**
3. For the satellite in Problem 4 above, let and. Let the satellite’s initial attitude be 0 degrees.
4. At, the attitude is commanded to 20 degrees. Show that the response is **(5-points)**
5. Repeating 5-A, show that for and that the satellite’s response is (**5-points)**
6. Verify the solution in 5-C by first checking the initial conditions and then substituting the solution into the system differential equation. Recall that with the values of and , one has from Problem 4-A: or (5-**points)**
7. For the satellite system in Problem 4, let the input be a step function in degrees. The satellite’s angle (t) will thus vary sinusoidally at 10 cycles per minute. Show that the amplifier gain and moment of inertia would be related as **(5-points)**