**Homework – Block Diagrams with Units**

1. Below is an antenna positioning system. Here, consider the control system for the yaw angle. Suppose that the power amplifier gain is and that the gear ratio and the angle sensor (the shaft encoder and data hold) are such that. The units for is volts and is degrees. Let be the input voltage to the motor. The transfer function of the motor pedestal is given by.





1. With the system open loop , a unit step function of voltage is applied to the motor []. Show that the steady-state output angle is degrees. Show that the steady-state output angular velocity is **(5-points)**
2. Using values stated in the problem, show that the transfer function with the gains and plant is the figure below **(5-points)**



1. Show that the block diagram changes to the one below when is in radians **(5-points)**



1. Given a step input of is applied at show that the response is **(5-points)**
2. Show that the response in (D) reaches steady-state in approximately 2 seconds **(5-points)**
3. Let the state variable model of a servomotor, given below, be equal to the transfer function of the motor pedestal (from Problem 1 above):

It can be shown that the solution for the above is. Also, from the block diagram of Problem 1-B, one has . Let and. Show that the state space realization is given as below **(5-points)**

1. Referring to Problem 1
2. Verify that the transfer function of Problem 1-B is given by , which yields the angle in degrees **(5-points)**
3. Show that the transfer function of 3-A such that is in radians is given by **(5-points)**
4. Verify the results of 3-B by showing the transfer function of 1-C is given by **(5-points)**
5. The figures below depict a satellite (left) and associated block diagram of the satellite’s attitude control system (right). Torque from the thrusters change the satellite’s yaw angle.





Since there is essentially no friction in space and assuming the satellite is rigid, then the moment of inertia about the yaw axis is related to the torque:

The Laplace transform yields:

Alternatively we have the plant transfer function:

1. Show that the transfer function **(5-points)**
2. Show that the transfer function of two systems in series (cascade), as seen below, is equal to the product of the transfer functions i.e. **(5-points)**



1. Show the transfer function for the following **(5-points each)**







1. A feedback control system is given below. The plant transfer function is



1. Show that the plant’s differential equation that relates and is given by. **(5-points)**
2. The compensator and sensor transfer functions are given by and. Modify the equation of part (A) to show that the differential equation that relates and is given by **(5-points)**
3. Show that the system transfer function from the results of part (B) is given by **(5-points)**
4. Use the relationship for block diagrams like the one above to show that **(5-points)**
5. The transfer function pole term yields a time constant where is real. Show that the time constant for the open-loop system is seconds and the time constant for the closed-loop system is seconds **(5-points)**
6. Repeat Question 3 but with the transfer functions, , and .
7. Show that the plant’s differential equation that relates and is given by. **(5-points)**
8. Modify the equation of part (A) to show that the differential equation that relates and is given by +16r **(5-points)**
9. Show that the system transfer function from the results of part (B) is given by **(5-points)**
10. Use the relationship for block diagrams like the one above to show that **(5-points)**
11. For part (e), recall that the transfer function’s underdamped pole terms yields a time constant. Show that the time constant for the open-loop system is second and the time constant for the closed-loop system is seconds **(25-points)**.
12. Repeat Question 3 but with the transfer functions, , and **(25-points)**.
13. Show that the plant’s differential equation that relates and is given by. **(5-points)**
14. Modify the equation of part (A) to show that the differential equation that relates and is given by **(5 points)**
15. Show that the system transfer function from the results of part (B) is given by **(5-points)**
16. Use the relationship for block diagrams like the one above to show that **(5-points)**
17. Show that the time constant for the open-loop system is second and the time constants for the closed-loop system is milliseconds and seconds **(25-points)**.