**Homework – Block Diagrams with Units**

1. Show the following
2. Show that the transfer function of two systems in parallel, as seen below, is equal to the sum of the transfer functions i.e. $C\left(s\right)=\left(G\_{1}+G\_{2}\right)E(s)$ **(5-points)**



1. Show that the transfer function of two systems in series (cascade), as seen below, is equal to the product of the transfer functions i.e. $C\left(s\right)=G\_{2}G\_{1}E(s)$ **(5-points)**



1. Show the transfer function $C(s)/R(s)$ for the following **(5-points each)**



1. $\frac{C(s)}{R(s)}=\frac{G\_{c}G\_{p}}{1+G\_{c}G\_{p}H}$



1. $\frac{C(s)}{R(s)}=\frac{G\_{1}G\_{3}+G\_{2}G\_{3}}{1+HG\_{1}G\_{3}}$
2. $\frac{C(s)}{R(s)}=\frac{G\_{1}G\_{2}}{1+H\_{2}G\_{1}+H\_{1}G\_{1}G\_{2}}$



1. A feedback control system is given below. The plant transfer function is $G\_{p}\left(s\right)=\frac{5}{0.2s+1}$



1. Show that the plant’s differential equation that relates $c(t)$ and$ m(t)$ is given by$ \dot{c}\left(t\right)+5c\left(t\right)=25m(t)$. **(5-points)**
2. The compensator and sensor transfer functions are given by $G\_{c}\left(s\right)=10$ and$ H\left(s\right)=1$. Modify the equation of part (A) to show that the differential equation that relates $c\left(t\right)$and$ r(t)$ is given by $\dot{c}\left(t\right)+255c\left(t\right)=250r(t)$ **(5-points)**
3. Show that the system transfer function from the results of part (B) is given by $\frac{C(s)}{R(s)}=\frac{250}{s+255}$ **(5-points)**
4. Use the relationship $\frac{C(s)}{R(s)}=\frac{G\_{c}G\_{p}}{1+G\_{c}G\_{p}H}$ for block diagrams like the one above to show that $\frac{C(s)}{R(s)}=\frac{250}{s+255}$ **(5-points)**
5. The transfer function pole term $ (s+a)$ yields a time constant $ τ=1/a$ where $ a$ is real. Show that the time constant for the open-loop system is $ τ=0.2$ seconds and the time constant for the closed-loop system is $τ=3.92$ seconds **(5-points)**
6. Repeat Question 3 but with the transfer functions$ G\_{c}=2$, $ G\_{p}=\frac{3s+8}{s^{2}+2s+2}$, and $ H\left(s\right)=1$.
7. Show that the plant’s differential equation that relates $c(t)$ and$ m(t)$ is given by$ \ddot{c}\left(t\right)+2\dot{c}\left(t\right)+2c\left(t\right)=3\dot{m}\left(t\right)+8m(t)$. **(5-points)**
8. Modify the equation of part (A) to show that the differential equation that relates $c\left(t\right)$and$ r(t)$ is given by $ \ddot{c}+8\dot{c}+18c=6\dot{r}$+16r **(5-points)**
9. Show that the system transfer function from the results of part (B) is given by $\frac{C(s)}{R(s)}=\frac{6s+16}{s^{2}+8s+18}$ **(5-points)**
10. Use the relationship $\frac{C(s)}{R(s)}=\frac{G\_{c}G\_{p}}{1+G\_{c}G\_{p}H}$ for block diagrams like the one above to show that $\frac{C(s)}{R(s)}=\frac{6s+16}{s^{2}+8s+18}$ **(5-points)**
11. For part (e), recall that the transfer function’s underdamped pole terms $ [\left(s+a\right)^{2}+b^{2}]$ yields a time constant$ τ=1/a$. Show that the time constant for the open-loop system is $ τ=1$ second and the time constant for the closed-loop system is $τ=0.25$seconds **(25-points)**.
12. Repeat Question 3 but with the transfer functions$ G\_{c}=2$, $ G\_{p}=\frac{5}{s^{2}+2s+2}$, and $ H\left(s\right)=3s+1$ **(25-points)**.
13. Show that the plant’s differential equation that relates $c(t)$ and$ m(t)$ is given by$ \ddot{c}\left(t\right)+2\dot{c}\left(t\right)+2c\left(t\right)=3\dot{m}\left(t\right)+8m(t)$. **(5-points)**
14. Modify the equation of part (A) to show that the differential equation that relates $c\left(t\right)$and$ r\left(t\right)$ is given by $ \ddot{c}+32\dot{c}+12c=10r$**(5 points)**
15. Show that the system transfer function from the results of part (B) is given by $\frac{C(s)}{R(s)}=\frac{10}{s^{2}+32s+12}$ **(5-points)**
16. Use the relationship $\frac{C(s)}{R(s)}=\frac{G\_{c}G\_{p}}{1+G\_{c}G\_{p}H}$ for block diagrams like the one above to show that $\frac{C(s)}{R(s)}=\frac{10}{s^{2}+32s+12}$ **(5-points)**
17. Show that the time constant for the open-loop system is $ τ=1$ second and the time constants for the closed-loop system is $τ\_{1}=31.63$ milliseconds and $τ\_{2}=2.63$ seconds **(25-points)**.