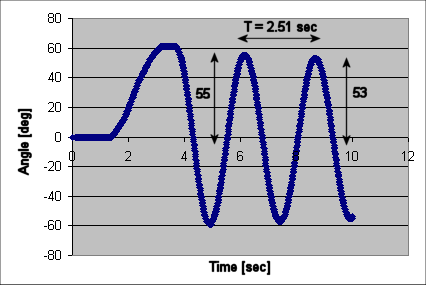
**Homework – Poles and Zeros**

1. The differential equation for a 2nd order system is expressed as. Here, are the natural frequency and damping ratio respectively.
2. Derive the roots of this equation **(5-points)**
3. Derive the solution  where are some constants **(10-points)**
4. Below is a free-body diagram (left) of the damped compound pendulum. Dimensions and experimental data plotting the time response from a free-fall experiment is also shown (right)

|  |  |  |  |
| --- | --- | --- | --- |
|  | Bar length | 0.495 |  |
|  | Pivot to CG distance | 0.023 |  |
|  | Mass of pendulum | 0.43 |  |

..\..\figures\simplePlankFbd010905.wmf

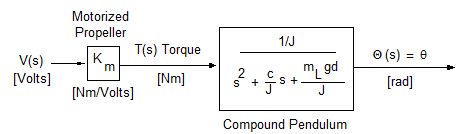


The logarithmic decrement equation states that where are the magnitudes of the and peaks. These peaks are related by the relationship where is the time in seconds between these peaks

1. Show that  and **(5-points)**.
2. Show that with these values that the roots are **(5-points)**
3. Sketch these roots on the imaginary plane. Show that and that ) and that **(10-points)**
4. Given the following block diagram and values, show that the open-loop transfer function is given by **(5 points)**



|  |  |  |
| --- | --- | --- |
|  | = 0.0090 |  |
|  | = 0.00035 |  |
|  | = 0.43 |  |
|  | = 0.023 |  |
|  | = 9.81 |  |
|  | = 0.017 |  |



1. Show that the time constant and settling time for the damped compound pendulum given that and natural frequency **(5 points)**
2. Show that the complex poles will yield and phase angle **(5 points)**