**Homework – Free Body and 2nd Order Systems**



Figure 1: Free-body Diagram of Damped Compound Pendulum (DCP)

1. Given the DCP in Figure 1, derive that $\ddot{θ}= \frac{c}{J}\dot{θ}+\frac{m\_{L}gd}{J}θ=0$ where $J $is the moment of inertia in$[kg∙m^{2}]$, $c $is the viscous damping coefficient in $[\frac{Nms}{rad}]$, $m\_{L} $is the pendulum mass in$[kg]$, $d $is the pivot distance to the center of gravity in $\left[m\right]$and $L $is the pendulum length in$[m]$. What assumption(s) are you making to derive this equation? **(10-points)**
2. Show that the natural frequency$ω\_{n}=\sqrt{\frac{m\_{L}gd}{J}}$. What are the units? **(10-points)**
3. Show that the viscous damping coefficient $c=2ζω\_{n}J$ where $ζ$is the damping ratio. What are the units? **(10-points)**

Below is a free-body diagram of the damped compound pendulum, dimensions and experimental data plotting the time response from free fall.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Bar length | 0.495 |  |
|  | Pivot to CG distance | 0.023 |  |
|  | Mass of pendulum | 0.43 |  |





1. If is the viscous damping coefficient, show that the equation of motion for the damped compound pendulum sketch above is given the equation below **(10 points)**



1. Given the following block diagram and values, show that the open-loop transfer function is given by **(5 points)**



|  |  |  |
| --- | --- | --- |
|  | = 0.0090 |  |
|  | = 0.00035 |  |
|  | = 0.43 |  |
|  | = 0.023 |  |
|  | = 9.81 |  |
|  | = 0.017 |  |



1. Show that the time constant and settling time for the damped compound pendulum given that and natural frequency **(5 points)**
2. Show that the complex poles will yield and phase angle **(5 points)**