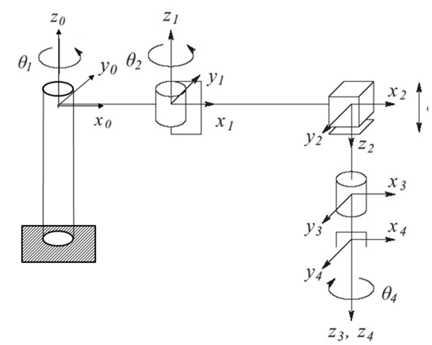
Homework – Inverse Kinematics

In lab and lecture, the inverse kinematics of a 2-DOF planar manipulator were derived and demonstrated.

1. Reconfigure the original 2-DOF planar manipulator such that Link 1 is a Beam 7 and Link 2 is a Beam 9 and appropriately change in **xl320-ik-1_0.nxc**. Affix four green-colored 1-stud bricks at points that reflect $(\theta_1, \theta_2) = (90, -90), (0, 90), (0, -90)$, and (-90,-90) degrees, where $\theta_1$and θ_2 are angles of Link 1 and Link 2 respectively.

   A. URL to your YouTube video demonstration (20-points)
   B. All files (e.g. NXC and Headers). Comment and make readable i.e. make good use of white space (10-points)

2. Recall the DH parameters for the SCARA arm given below and you previously derived ${T_4}^0$.



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	${\theta^*}_1$
2	a_2	180	0	${\theta^*}_2$
3	0	0	${d^*}_3$	0
4	0	0	d_4	${\theta^*}_4$

* denotes variable

$$ {T_4}^0 = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The general problem of inverse kinematics is given a homogeneous transformation $H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix}$ find a solution (possibly non-unique) for $T_n^0(q_1 \cdots q_n) = H$ where $T_n^0(q_1 \cdots q_n) = A_1(q_1) \cdots A_n(q_n)$. In other words, $H$is the desired end-effector pose and one needs to find joint variables $q_1 \cdots q_n$so that $T_n^0(q_1 \cdots q_n) = H$. If say that ${T_4}^0 = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix}$ where $o = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ -d_3 - d_4 \end{bmatrix}$ is the end-effector position and $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is the end-effector orientation.

A. Use the figure below and the Law of Cosines to show that $\theta_2 = \tan^{-1}\left(c_2, \pm\sqrt{1 - c_2}\right)$ where $c_2 = \frac{{o_x}^2 + {o_y}^2 - {a_1}^2 - {a_2}^2}{2a_1a_2}$ (5-points)

B. Show that $\theta_1 = \tan^{-1}(o_x, o_y) - \tan^{-1}(a_1 + a_2c_2, a_2s_2)$ (5-points)

C. Given that $\theta_1 + \theta_2 - \theta_4 = \alpha = \tan^{-1}(r_{11}, r_{12})$ show that $\theta_4 = \theta_1 + \theta_2 - \alpha$ (5-points)

D. Show that $d_3 = o_z + d_4$ (5-points)

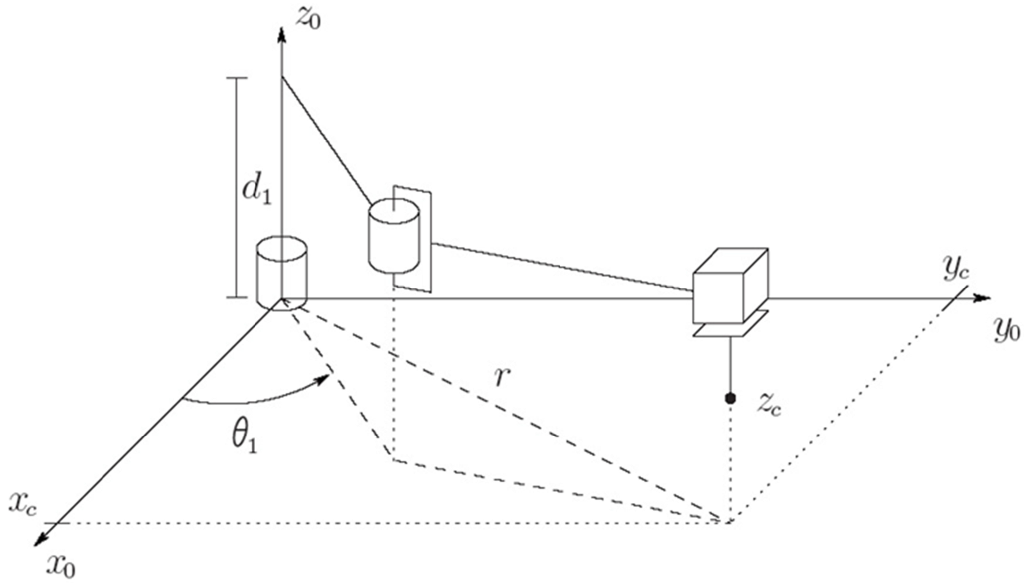


Figure: Configuring the SCARA as above, and projecting the manipulator onto the x_0y_0 plane shows that one can apply the Law of Cosines to show $c_2 = \frac{{o_x}^2+{o_y}^2-{a_1}^2-{a_2}^2}{2a_1a_2}$