**Homework – Inverse Kinematics**

In lab and lecture, the inverse kinematics of a 2-DOF planar manipulator were derived and demonstrated.

1. Reconfigure the original 2-DOF planar manipulator such that Link 1 is a Beam 7 and Link 2 is a Beam 9 and appropriately change in **xl320-ik-1\_0.nxc**. Affix four green-colored 1-stud bricks at points that reflect $\left(θ\_{1},θ\_{2}\right)=\left(90, -90\right), \left(0, 90\right), \left(0, -90\right), and (-90,-90) degrees$, where $θ\_{1}$and $θ\_{2}$ are angles of Link 1 and Link 2 respectively.
2. URL to your YouTube video demonstration (20-points)
3. All files (e.g. NXC and Headers). Comment and make readable i.e. make good use of white space (10-points)
4. Recall the DH parameters for the SCARA arm given below and you previously derived$ T\_{4}^{0}$.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Link | $$a\_{i}$$ | $$α\_{i}$$ | $$d\_{i}$$ | $$θ\_{i}$$ |
| 1 | $$a\_{1}$$ | 0 | 0 | $$θ^{\*}\_{1}$$ |
| 2 | $$a\_{2}$$ | 180 | 0 | $$θ^{\*}\_{2}$$ |
| 3 | 0 | 0 | $$d^{\*}\_{3}$$ | 0 |
| 4 | 0 | 0 | $$d\_{4}$$ | $$θ^{\*}\_{4}$$ |

\* denotes variable

$$T\_{4}^{0}=\left[\begin{matrix}c\_{12}c\_{4}+s\_{12}s\_{4}&-c\_{12}s\_{4}+s\_{12}c\_{4}&0&a\_{1}c\_{1}+a\_{2}c\_{12}\\s\_{12}c\_{4}-c\_{12}s\_{4}&-s\_{12}s\_{4}-c\_{12}c\_{4}&0&a\_{1}s\_{1}+a\_{2}s\_{12}\\0&0&-1&-d\_{3}-d\_{4}\\0&0&0&1\end{matrix}\right]$$

The general problem of inverse kinematics is given a homogeneous transformation $H=\left[\begin{matrix}R&o\\0&1\end{matrix}\right]$ find a solution (possibly non-unique) for $T\_{n}^{0}\left(q\_{1}\cdots q\_{n}\right)=H$ where$ T\_{n}^{0}\left(q\_{1}\cdots q\_{n}\right)=A\_{1}(q\_{1})\cdots A\_{n}(q\_{n})$. In other words, $H$is the desired end-effector pose and one needs to find joint variables $q\_{1}\cdots q\_{n}$so that$ T\_{n}^{0}\left(q\_{1}\cdots q\_{n}\right)=H$. If say that $T\_{4}^{0}=\left[\begin{matrix}R&o\\0&1\end{matrix}\right] $where $o=\left[\begin{matrix}o\_{x}\\o\_{y}\\o\_{z}\end{matrix}\right]=\left[\begin{matrix}a\_{1}c\_{1}+a\_{2}c\_{12}\\a\_{1}s\_{1}+a\_{2}s\_{12}\\-d\_{3}-d\_{4}\end{matrix}\right]$ is the end-effector position and $ R=\left[\begin{matrix}r\_{11}&r\_{12}&r\_{13}\\r\_{21}&r\_{22}&r\_{23}\\r\_{31}&r\_{32}&r\_{33}\end{matrix}\right]=$ $\left[\begin{matrix}c\_{12}c\_{4}+s\_{12}s\_{4}&-c\_{12}s\_{4}+s\_{12}c\_{4}&0\\s\_{12}c\_{4}-c\_{12}s\_{4}&-s\_{12}s\_{4}-c\_{12}c\_{4}&0\\0&0&-1\end{matrix}\right]$ is the end-effector orientation.

1. Use the figure below and the Law of Cosines to show that $θ\_{2}=tan^{-1}\left(c\_{2},\pm \sqrt{1-c\_{2}}\right)$ where $c\_{2}=\frac{o\_{x}^{2}+o\_{y}^{2}-a\_{1}^{2}-a\_{2}^{2}}{2a\_{1}a\_{2}}$ (5-points)
2. Show that $θ\_{1}=tan^{-1}(o\_{x},o\_{y})-tan^{-1}(a\_{1}+a\_{2}c\_{2}, a\_{2}s\_{2}$) (5-points)
3. Given that $θ\_{1}+θ\_{2}-θ\_{4}=α=tan^{-1}\left(r\_{11}, r\_{12}\right)$ show that $θ\_{4}=θ\_{1}+θ\_{2}-α$ (5-points)
4. Show that $d\_{3}=o\_{z}+d\_{4}$ (5-points)



Figure: Configuring the SCARA as above, and projecting the manipulator onto the $x\_{0}y\_{0}$ plane shows that one can apply the Law of Cosines to show $c\_{2}=\frac{o\_{x}^{2}+o\_{y}^{2}-a\_{1}^{2}-a\_{2}^{2}}{2a\_{1}a\_{2}}$