Robotics

Forward Kinematics

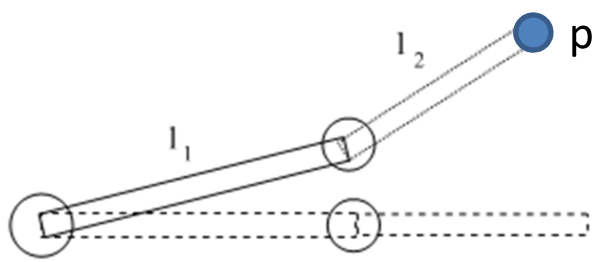
Reference: lectureDenavitHartenberg-103116.pptx
And lectureDenavitHartenberg-103116-011120a.pptx

Forward Kinematics

Big Question: Want robot's end-effector at desired point. What joint angles are needed?

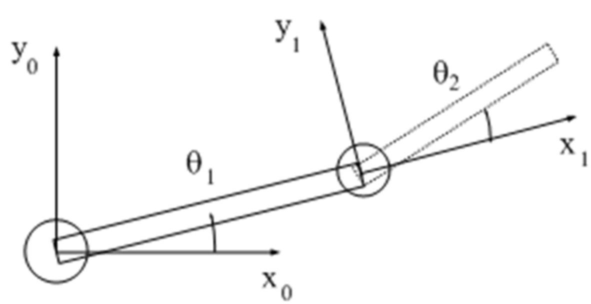
- Inverse Kinematics is the topic that answers this
- Solution (if any) likely is non-linear, transcendental, and often non-unique
- Demands inverting a (transformation) matrix – multiple techniques
- Solution begins by understanding the transformation matrix
- So must first understand Forward Kinematics

Sanity Check: Given the 2-link planar manipulator below, what is the (x, y) position of the end-point p?

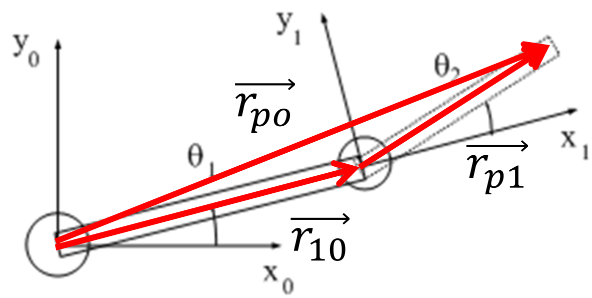


Solution: Recall Relative Reference Frames from Physics, Mechanics and Kinematics

Step 1: Assign a reference frame



Step 2: Apply vectors



$$\overrightarrow{r_{po}} = \overrightarrow{r_{10}} + \overrightarrow{r_{p1}}$$

Have the following:

$$\overrightarrow{r_{10}} = l_1 \cos\theta_1 \hat{\imath} + l_1 \sin\theta_1 \hat{\jmath}$$

$$\overrightarrow{r_{p1}} = l_2 \cos\theta_2 \widehat{i'} + l_2 \sin\theta_2 \widehat{j'}$$

Can show:

$$x_{po} = l_1 c_1 + l_2 c_{12}$$
$$y_{po} = l_1 s_1 + l_2 s_{12}$$

where

$$c_k = \cos\theta_k \text{ and } s_k = \sin\theta_k$$
$$c_{12} = \cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \quad (1)$$
$$s_{12} = \sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \sin\theta_2 \cos\theta_1$$

Take-away:

- The reference frames' poses are not unique – can position and orient anywhere
- Solving for absolute position for N-links, where N is large, is tedious

Homogeneous Transformation Matrix

- Assigning reference frames is non-unique
- But DH notation serves as a kind of "standard" by roboticists
- Much (robot) math (and simulators) builds upon using this DH "standard"
- Homogeneous Transformation Matrix and Tool Frame are examples

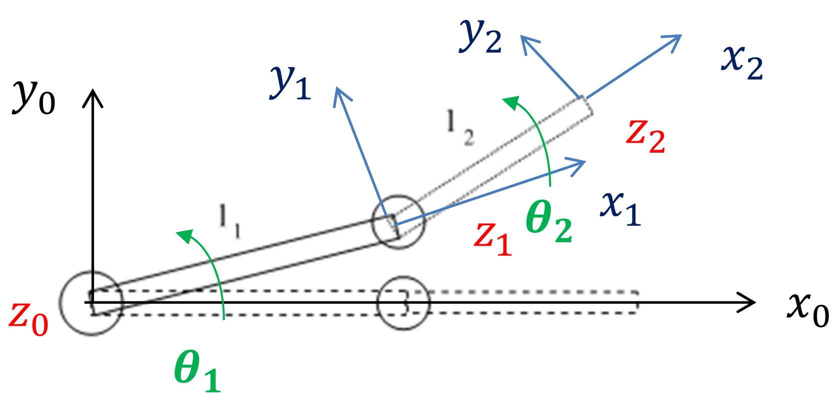
$$A_i \triangleq \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where A_i is called the Homogeneous Transformation Matrix and maps frame from $i-1$ to i

Tool Frame $\boldsymbol{T_n^0}$: More interested in knowing the transformation from robot's base (origin) to robot's end-effector (tool) frames

$$\boldsymbol{T_n^0 = A_1 A_2 \cdots A_n}$$

Sanity Check: Try homogeneous and tool transformation matrices with 2-link planar manipulator



Link k	θ_k	d_k	α_k	a_k
1	θ_1	0	0	l_1

Link k	θ_k	d_k	α_k	a_k
2	θ_2	0	0	l_2

Step 1: Solve individual homogeneous transformation matrices A_i

$$A_1 \triangleq \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A_2 \triangleq \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Footnote: hand-checked A1 and A2 10/31/16, 11/04/16; and 04/04/19.
Also see Spong pg. 84-85 Robot Modeling and Control 2006 Edition

Step 2: Solve product of homogeneous transformation matrices A_i

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $c_k = \cos\theta_k$ and $s_k = \sin\theta_k$

$$c_{12} = \cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

$$s_{12} = \sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \sin\theta_2 \cos\theta_1$$

Note: The last column gives (x, y) position of end-point. Compare to (1) from before

$$x_{po} = l_1 c_1 + l_2 c_{12}$$
$$y_{po} = l_1 s_1 + l_2 s_{12}$$

Takeaway: DH notation gives step-by-step analytical method to ultimately determine the (x, y) position of a tool (end-effector) with respect to base frame (i.e. forward kinematics)

Inverting the tool matrix, should enable one to determine the joint values needed to put tool at desired (x, y) and is called inverse kinematics which is topic of next lecture.

Footnote: hand-checked $T_2{}^0$ 10/31/16, 11/04/16; and 04/04/19.
Also see Spong pg. 84-85 Robot Modeling and Control 2006 Edition