# ME729 Advanced Robotics - Homogeneous Transformations Supplements

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# Rotation Transformation - Supplement

- Recall the following example and think why is the multiplication right to left (pre-multiplication).
- Given $u = 7i + 3j + 2k$, rotate it about $z$ axis with $90°$ and about $y$ axis with $90°$, and translate it along $4i - 3j + 7k$.

$$\text{Trans(4, -3, 7)Rot(y, 90)Rot(z, 90)} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 10 \\ 1 \end{bmatrix}$$
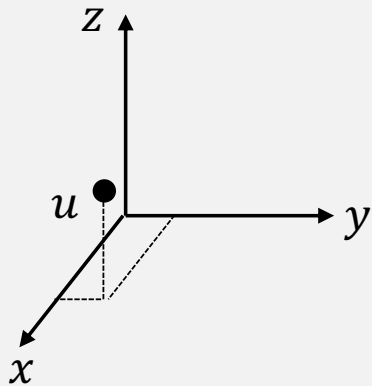
- There is an **u** vector.
- First, rotate **u** $90°$ about **z**. Set **v** as a transformed vector.
$$v = Rot(z, 90)u$$
- Second, rotate **v** $90°$ about **y**. Set **w** as a transformed vector.
$$w = Rot(y, 90)v$$
- Finally, translate **w** along $4i - 3j + 7k$. Set **p** as a transformed vector.
$$p = Trans(4, -3, 7)w$$
- Therefore,
$$p = Trans(4, -3, 7)Rot(y, 90)Rot(z, 90)u$$

# Rotation Transformation - Supplement

- The rotation and translation we have been describing have all been made w.r.t. the reference frame.
- Consider the example again, and we can interpret the transformation by two ways.

*Transformation w.r.t the reference frame.* ⟵

$$\text{Trans(4, -3, 7)Rot(y, 90)Rot(z, 90)}$$

⟶ *Transformation w.r.t the moving frame.*

1. Interpret the operation from right to left.
   - The frame is first rotated around the reference z axis by $90°$.
   - Then, rotated $90°$ around the reference y axis.
   - Finally, translated by $4i - 3j + 7k$.

2. Interpret the operation from left to right. → ***Relative transformation***
   - The object is first translated by $4i - 3j + 7k$.
   - Then, rotated $90°$ around the current frame axis, which in this case are the same as the reference axes.
   - Finally, rotated $90°$ about the newly rotated (current) frame axes.

- Pre-multiplication: translation and/ or rotation is made **w.r.t. the reference frame**.
- Post-multiplication: translation and/ or rotation is made **w.r.t. the moving frame**.