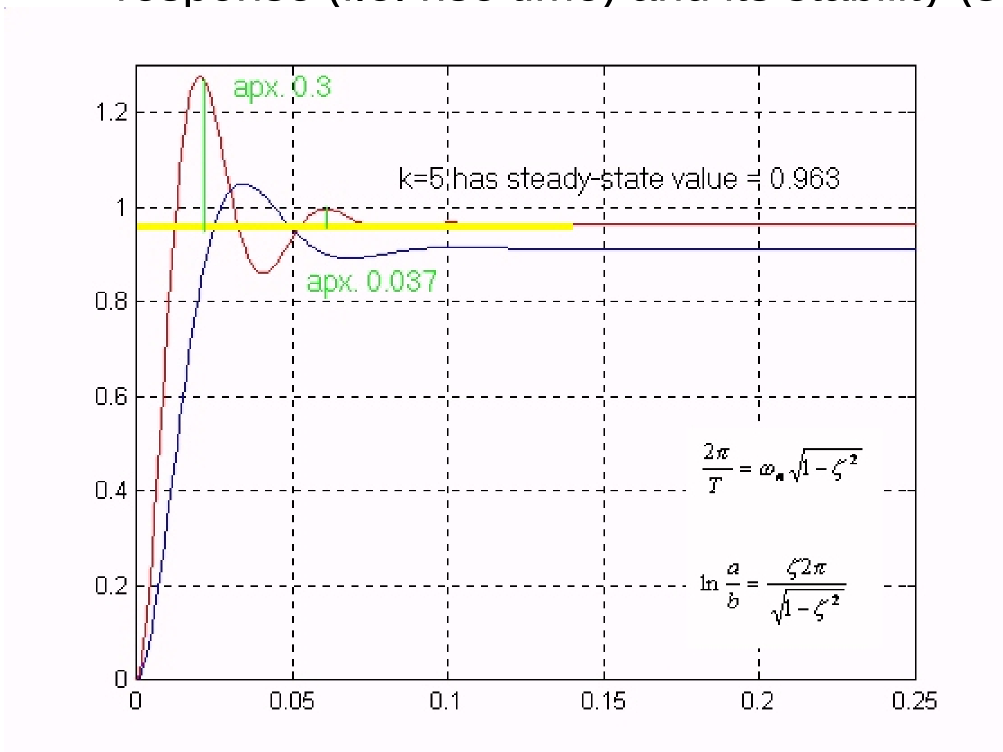
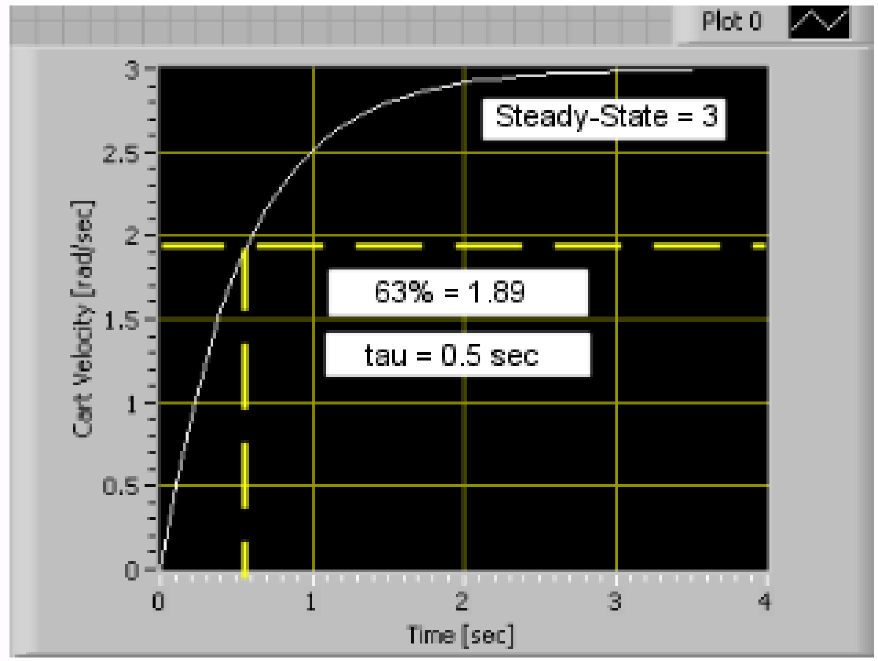
PID Case Study: Velocity Control of a Motorized Winch-and-Cart

Objective: System Identification

System ID attempts to capture a plant's characteristics such as transient response (i.e. rise time) and its stability (steady-state)





Typical 2nd Order Response

- Car shock absorbers
- Pneumatic pistons
- Economic systems

Typical 1st Order Response

- DC motors
- Damped hydraulics

Grandmother Explanation?

Lego-Based Motorized Winch-and-Cart

Control Goal: Winch load at constant velocity

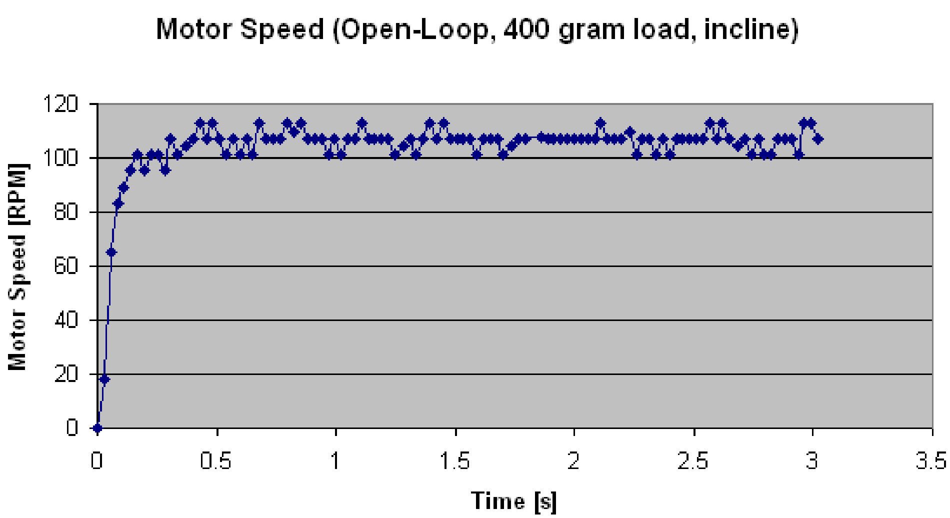
Practical Applications:

- Tow truck
- Jeep winches
- Elevators

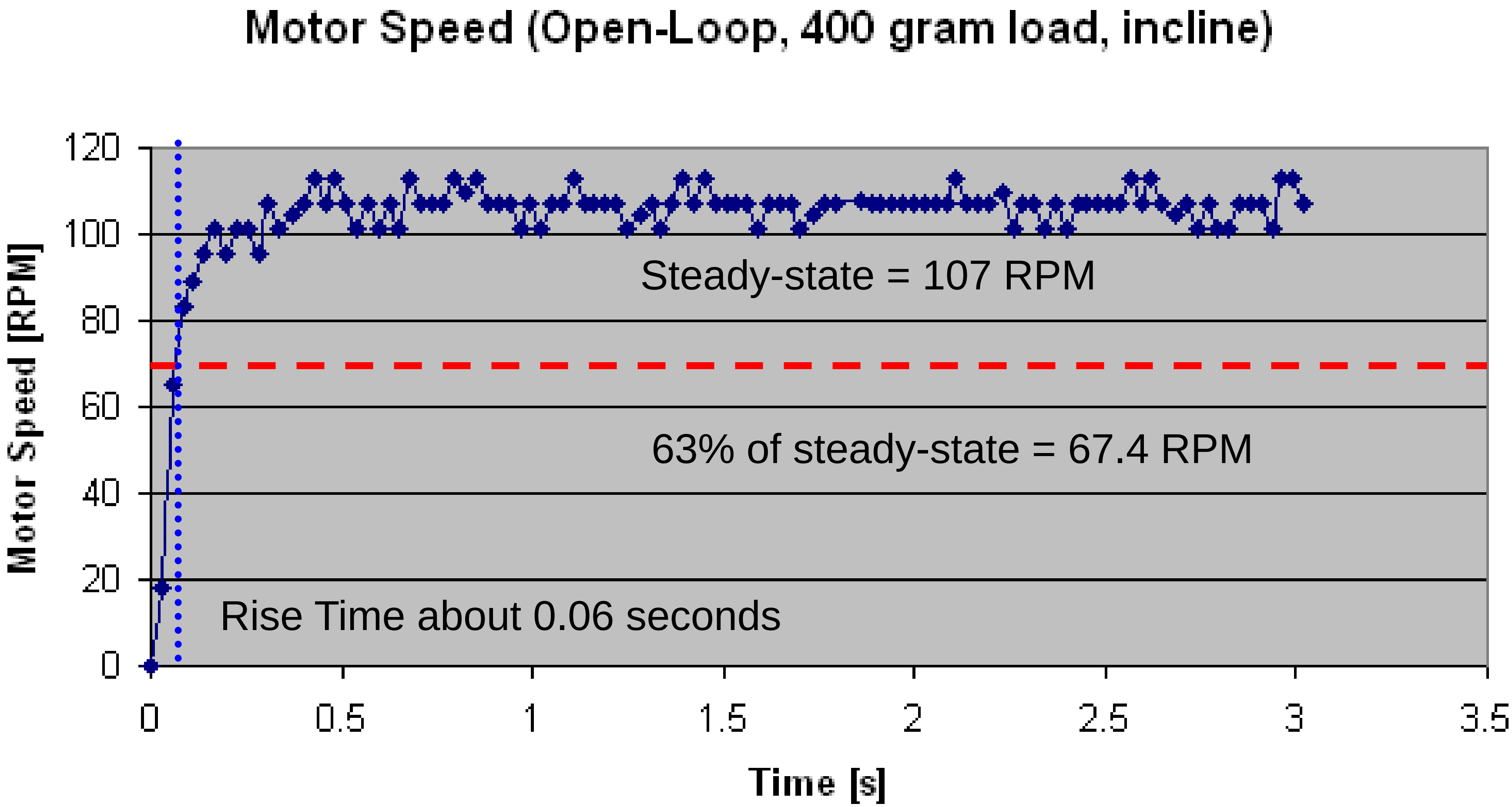
Winch at constant velocity regardless of load

Step 1: Need to characterize open-loop system

Recall: motorized winch-and-cart max power occurs for 400 gram mass (0.12 Nm)
For inclined surface, winch speed will be faster. Need step response



- NXT commanded at 100% level
- Steady state velocity about 107 RPM
- 63% of 107 RPM = 67.4 RPM



NB: Want sampling time about 10 to 20 times faster than rise time.
Hence sampling time should be about 3 to 6 milliseconds

Brick Motor Level input $f(t)$ to the cart motor, yields velocity output $y(t)$

Consequently have:

$$\tau \frac{dy}{dt} + y = k \cdot f(t) \tag{1}$$

where τ is the time constant and k is the steady-state gain (to be determined)

The Laplace form of (1) yields:

$$Y(\tau s + 1) = k \cdot F \tag{2}$$

The input-output transfer function becomes

$$\frac{Y(s)}{F(s)} = \frac{k}{1 + \tau s} \tag{3}$$

Also, the solution to the first-order differential equation given in (1) is

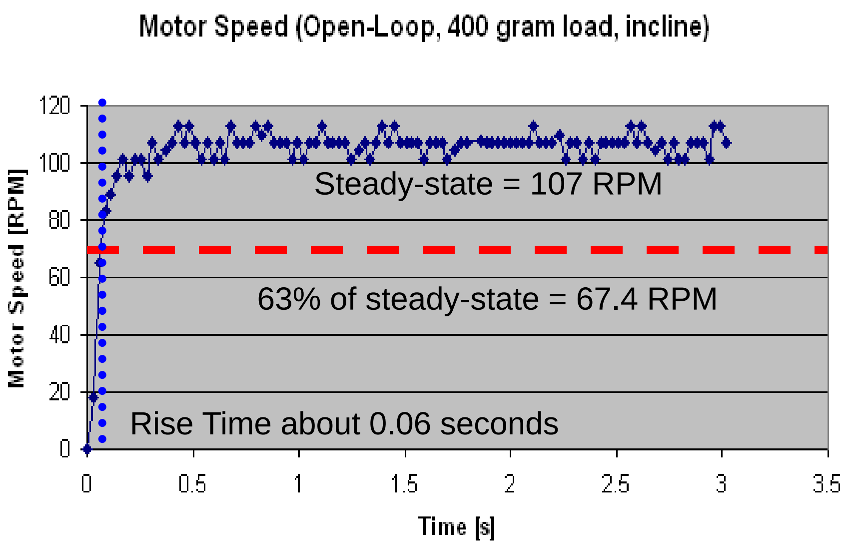
$$y(t) = Y_{ss}\left(1 - e^{-t/\tau}\right) \tag{4}$$

To calculate the steady-state gain, apply a step input $f(t) = Au(t)$

where A is a constant

At steady-state, with step input, (1) becomes:

$$\tau \cancelto{0}{\frac{dy}{dt}} + \cancelto{Y_{ss}}{y} = k \cdot f(t) \quad \text{or} \quad k = \frac{Y_{ss}}{A} = \frac{\text{Steady State Velocity [RPM]}}{\text{Step Input Level [Motor \%]}} \qquad (5)$$



Step response for $A = 100\%$ Motor Level

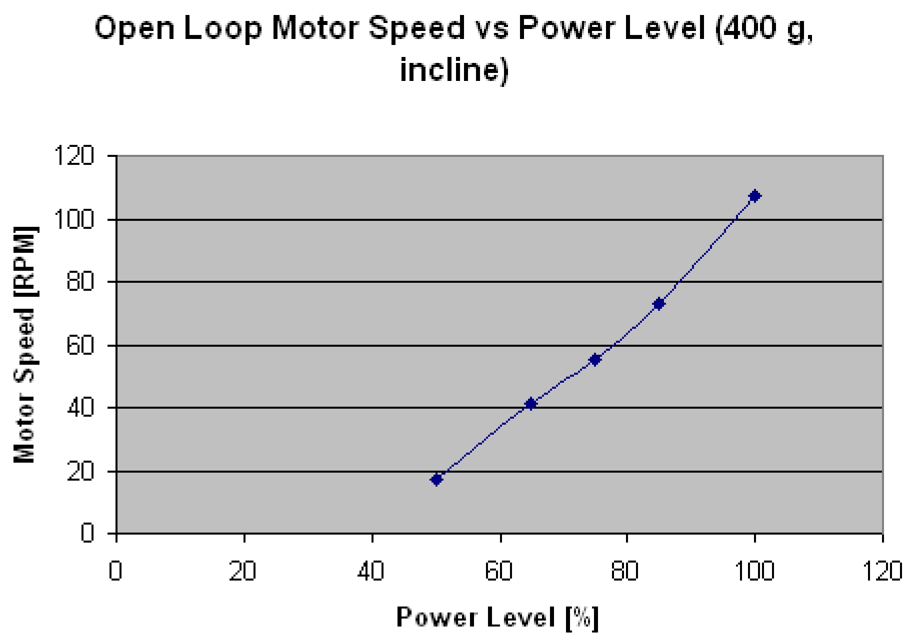
Eyeballing, observe that:

$$Y_{ss} = 107 \text{ RPM}$$

Hence:

$$k = \frac{Y_{ss}}{A} = \frac{107 \text{ RPM}}{100\% \text{ Level}} = 1.07 \text{ RPM/Level}$$

Power Level [%]	Steady-State Speed [RPM]
100	107
85	73
75	55
65	41
50	17



$$\frac{\text{Motor RPM}}{\text{Power Level}} = \frac{107-41}{100-65} = \frac{66}{35} = 1.89$$

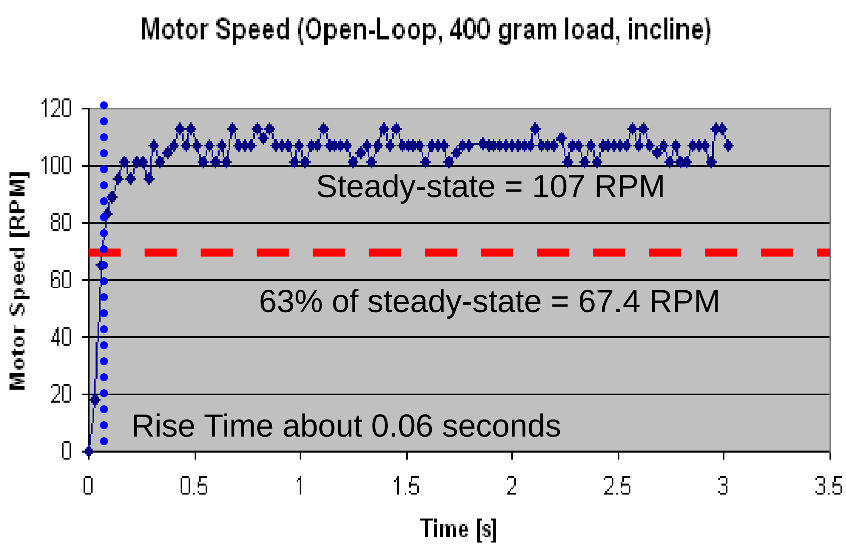
Blah: check this slope calculation…

Recall (4)

$$y(t) = Y_{ss}\left(1 - e^{-t/\tau}\right)$$

at $t = \tau$

$$y(\tau) = Y_{ss}\left(1 - e^{-1}\right) = 0.632 \cdot Y_{ss}$$



63% value is about $Y = 67.4$ RPM

Hence

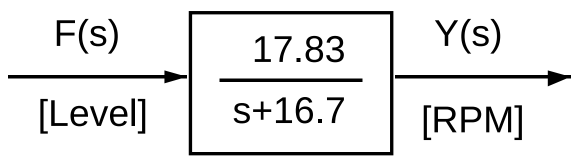
$$\tau = 0.06\,\text{sec}$$

Thus from (3) open-loop transfer function due to step input of 100% motor level yields:

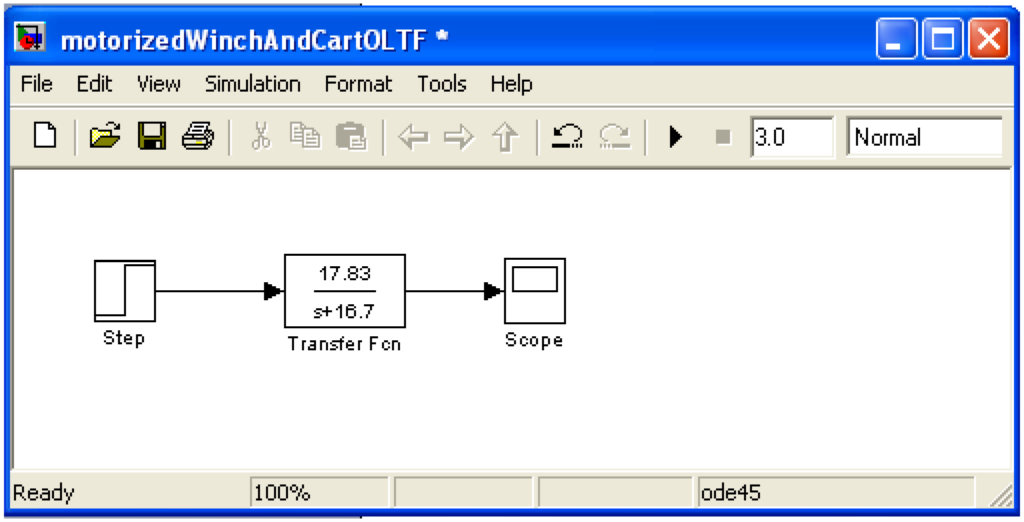
$$\frac{Y(s)}{F(s)} = \frac{k}{1+\tau s} \quad \text{where} \quad \tau = 0.06\,\text{sec} \quad \text{and} \quad k = 1.07\,\text{RPM/Level}$$

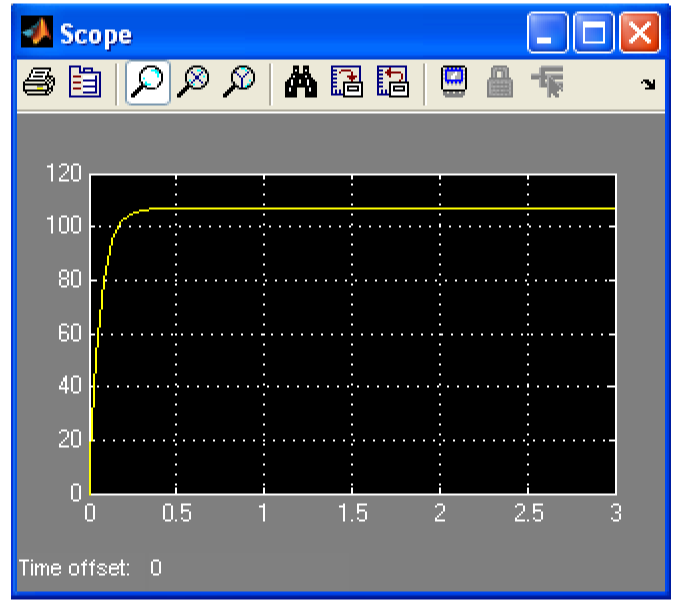
$$\frac{Y(s)}{F(s)} = \frac{1.07}{1+0.06s} = \frac{1.07}{0.06\left(s+\frac{1}{0.06}\right)} = \frac{17.83}{s+16.7} \qquad (6)$$

The block diagram representing (6) is simply:

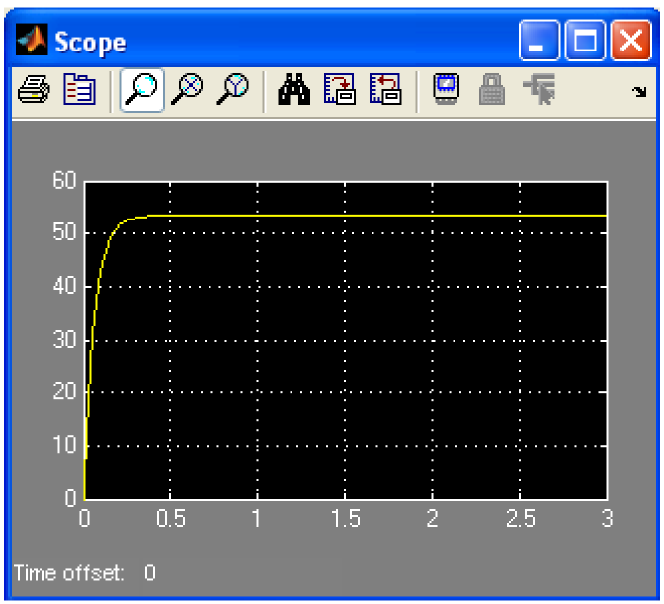


Simulink of (6)



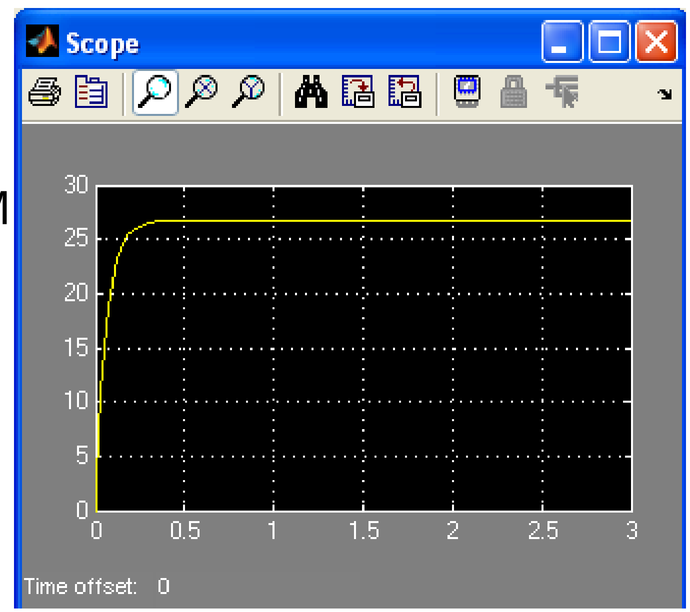


100% motor level (400 g, incline)



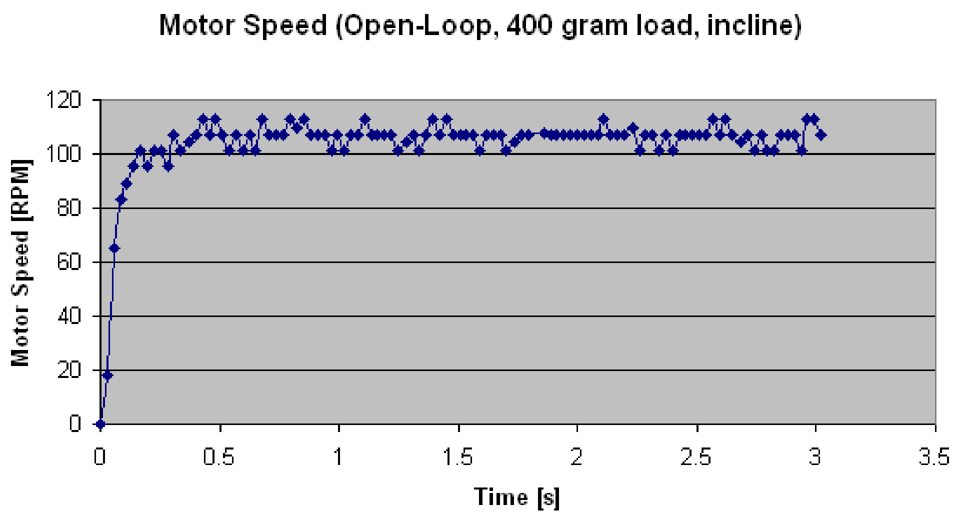
~53 RPM

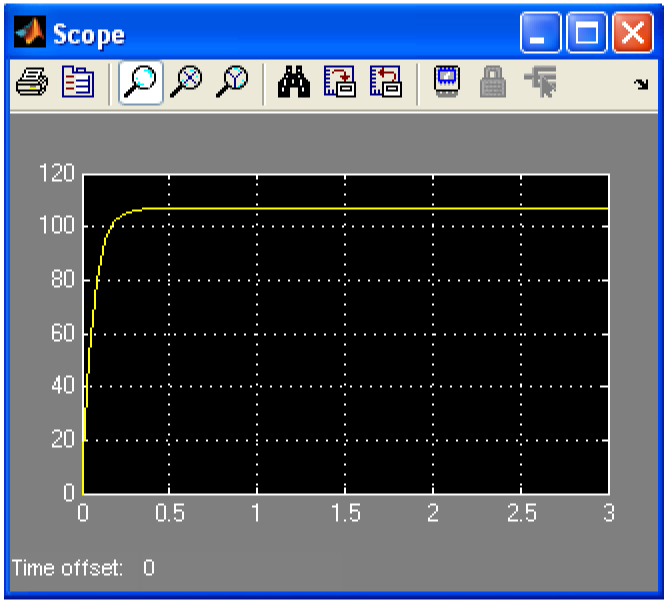
50% motor level (400 g, incline)



~27 RPM

25% motor level (400 g, incline)





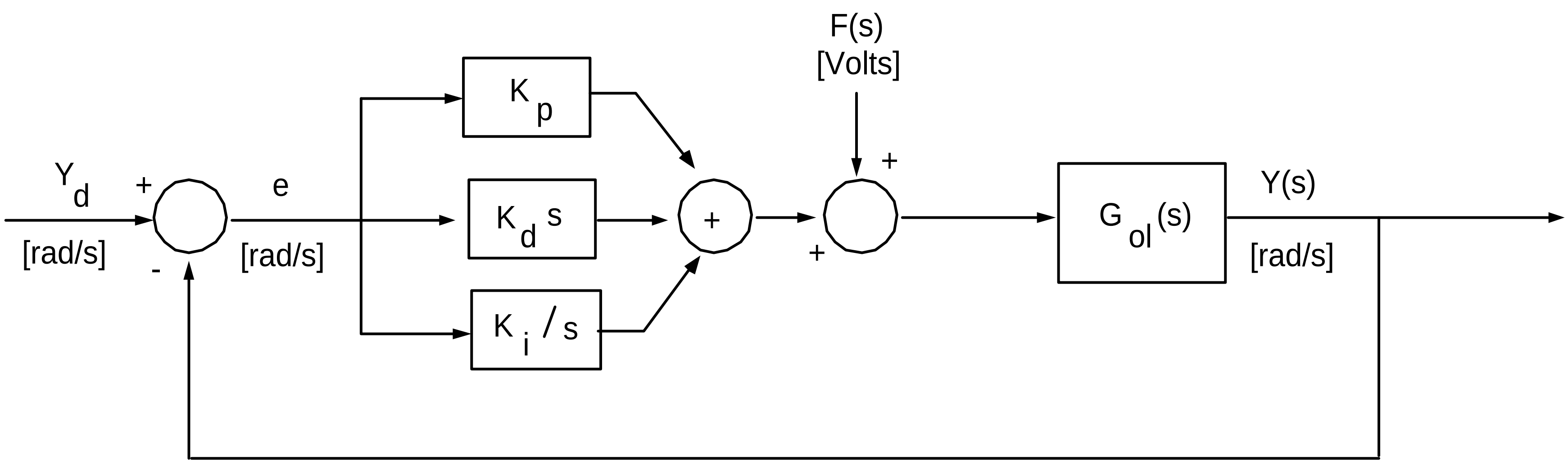
Note similarity between experimental and simulated plots

PID Control (Closed-Loop) of Motorized Winch-and-Cart

Goal: Want cart velocity to always be 63 RPM

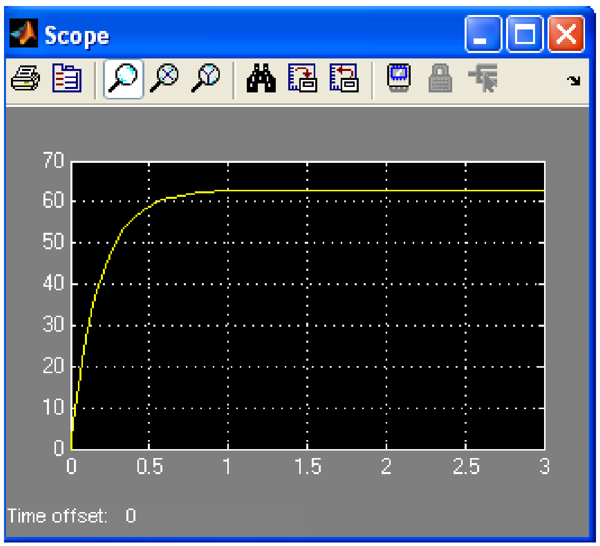
- Even if load changes (within motor limits)
- Even if Brick voltage changes (within limits)

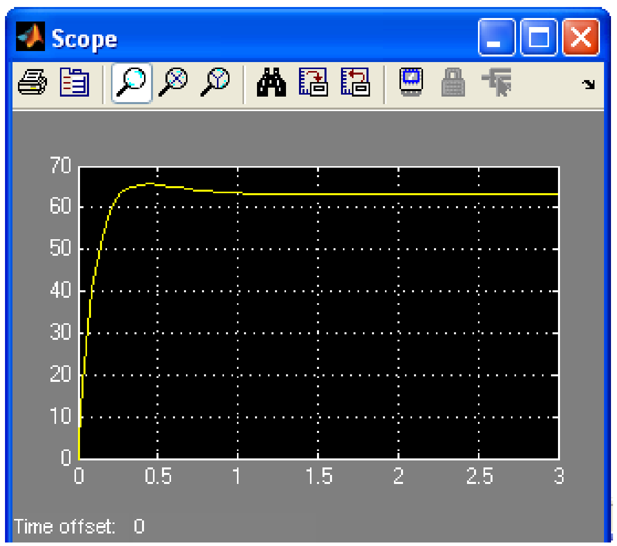
PID is the most common form of closed-loop control:

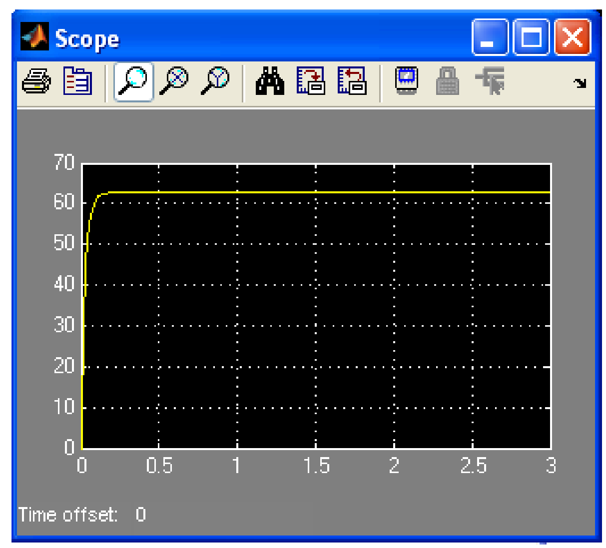


Why?

Answer: One can tune for desired performance without full knowledge of dynamics





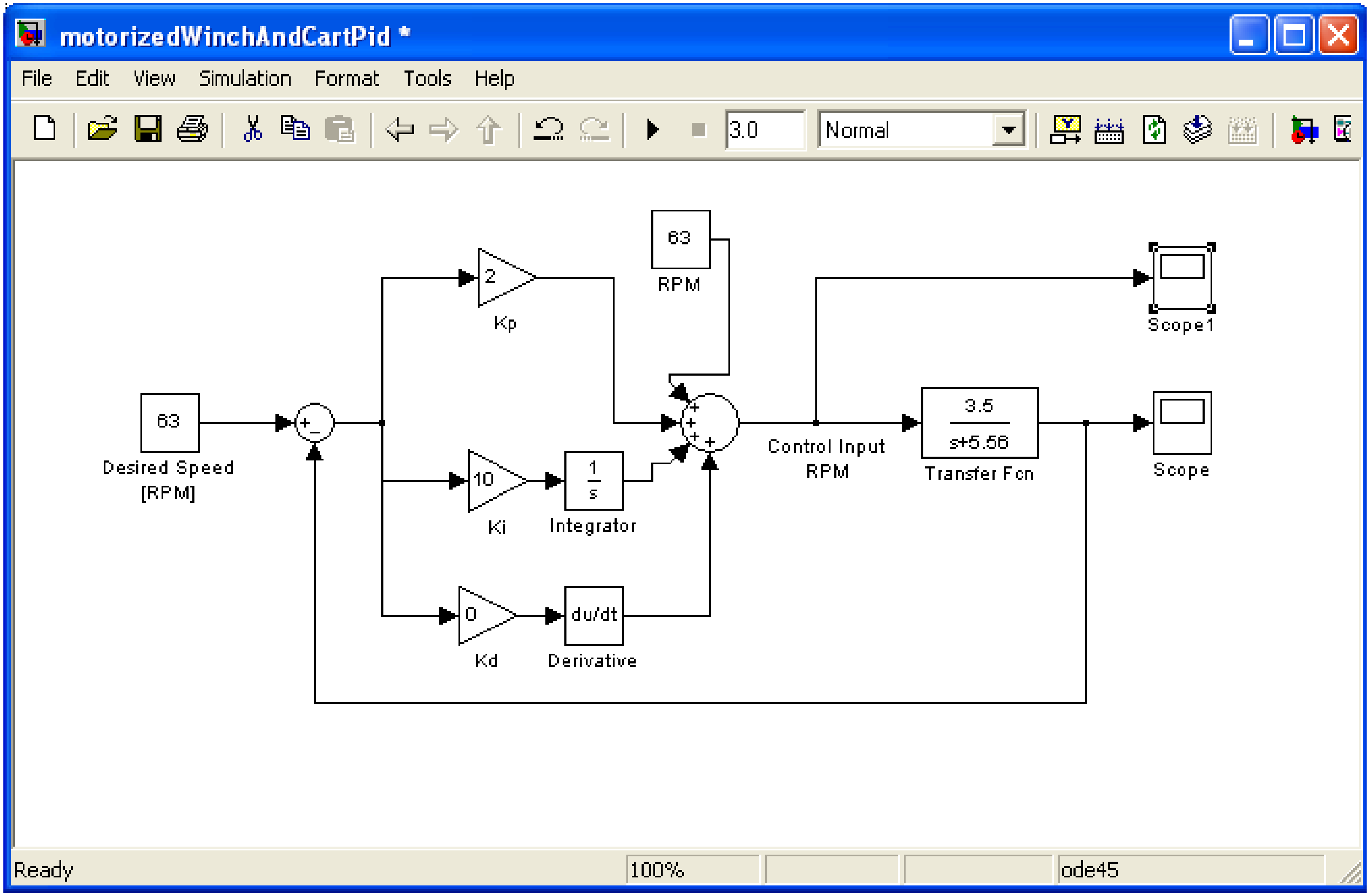


Without Control
(i.e. Open-loop)

$K_p = 2 \quad K_i = 10 \quad K_d = 0$

$K_p = 10 \quad K_i = 20 \quad K_d = 0$

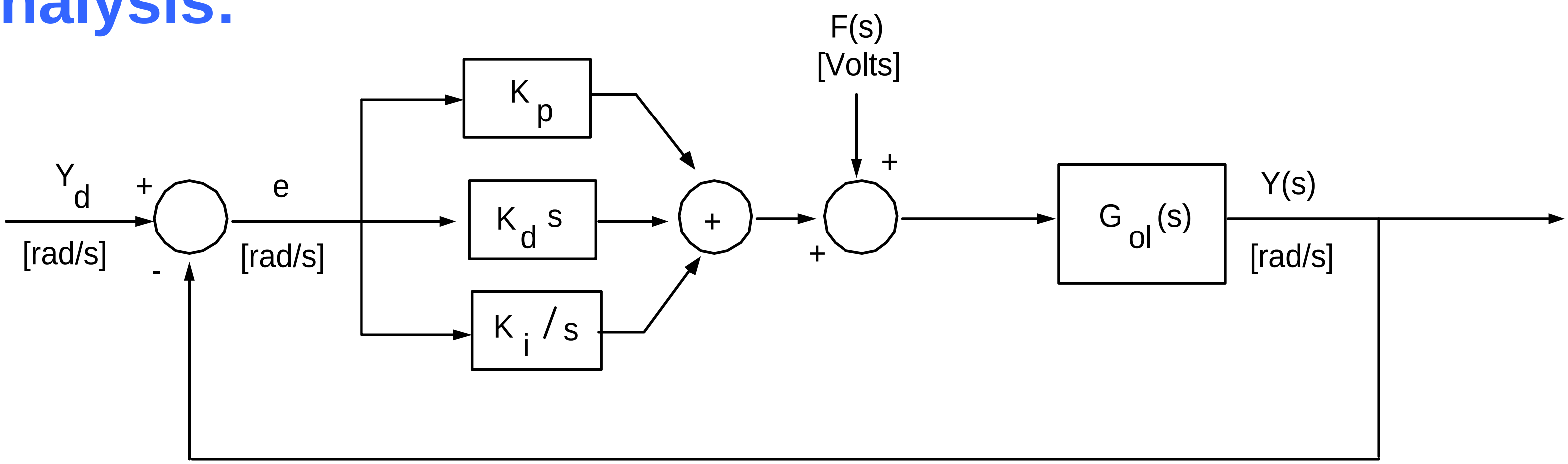
PID Simulink Simulation



May need to multiply desired and set point values to 100/63

However, tuning can be very tedious. Some knowledge of system type aids in tuning, make performance expectations realistic and avoid instability.

Analysis:



Input-Output Relationship given by:

$$Y(s) = G_{\mathrm{ol}} \left\{ F(s) + \left(K_p + \frac{K_i}{s} + K_d\, s \right) \left(Y_d - Y \right) \right\} \qquad (1)$$

Can reduce to show that (1) becomes:

$$Y(s) = \frac{G_{\text{ol}} s}{s + G_{\text{ol}}\left(K_p s + K_i + K_d s^2\right)} F + \frac{G_{\text{ol}}\left(K_p s + K_i + K_d s^2\right)}{s + G_{\text{ol}}\left(K_p s + K_i + K_d s^2\right)} Y_d \qquad (2)$$

Case Study 1: Proportional only control $Y_d = \frac{A}{s}$ So (2) becomes:

$$Y(s) = \frac{G_{\text{ol}} s}{s + G_{\text{ol}}\left(K_p s\right)} F + \frac{G_{\text{ol}}\left(K_p s\right)}{s + G_{\text{ol}}\left(K_p s\right)} \cdot \frac{A}{s} \qquad (3)$$

Final Value Theorem states that:

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s)$$

Thus steady-state part of (3) becomes:

$$y_{\text{ss}} = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} \frac{G_{\text{ol}}\left(K_p s\right)}{s + G_{\text{ol}}\left(K_p s\right)} A \qquad (4)$$

Given that cart velocity has an open-loop transfer function of the following form:

$$G_{\text{ol}}(s) = \frac{b}{s+a} \tag{5}$$

Substitution of (5) into (4) yields

$$y_{\text{ss}} = \lim_{s\to 0} \frac{\frac{b}{s+a}\left(K_p s\right)}{s + \frac{b}{s+a}\left(K_p s\right)} A = \lim_{s\to 0} \frac{bK_p s}{s(s+a)+bK_p s} A$$

Applying L'Hopital to calculate the limit yields:

$$y_{\text{ss}} = \lim_{s\to 0} \frac{bK_p}{2s+a+bK_p} A = \frac{bK_p}{a+bK_p} A$$

Note: If K_p is very large, then $y_{ss} \approx A$

Will always have steady-state error

Case Study 2: Proportional + Integral control $Y_d = \frac{A}{s}$

So (2) becomes:

$$Y(s) = \frac{G_{\text{ol}} s}{s + G_{\text{ol}}\left(K_p s + K_i\right)} F + \frac{G_{\text{ol}}\left(K_p s + K_i\right)}{s + G_{\text{ol}}\left(K_p s + K_i\right)} \cdot \frac{A}{s}$$

Apply Final Value Theorem:

$$y_{\text{ss}} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot Y(s) = \frac{G_{\text{ol}}\left(K_p s + K_i\right)}{s + G_{\text{ol}}\left(K_p s + K_i\right)} A \qquad (6)$$

Substituting the OLTF (2) into (6) yields:

$$y_{\text{ss}} = \lim_{s\to 0} \frac{\frac{b}{s+a}\left(K_p s + K_i\right)}{s + \frac{b}{s+a}\left(K_p s + K_i\right)} A = \lim_{s\to 0} \frac{b\left(K_p s + K_i\right)}{s(s+a) + b\left(K_p s + K_i\right)} A = \frac{bK_i}{bK_i} A = A$$

Hence integral action ensures zero steady-state error

Systems like the motorized cart are called Type 0 systems:

$$G_{\mathrm{ol}}(s) = \frac{b}{s+a}$$

All transfer functions can be factored into the general form:

$$G_{ol}(s) = \frac{(\tau_a s+1)\cdots(\tau_m s+1)}{\boxed{s^i}(\tau_1 s+1)\cdots(\tau_n s+1)}$$

$i = 0$	Type 0 System
$i = 1$	Type 1 System
$i = 2$	Type 2 System

General form any TF

A Type i system is the number of “free” integrators, i. The motorized tethered cart, for velocity control, is a Type 0 system

For Type 0 Systems:

- Will always have steady-state error (with proportional only control): see Case 1
- Integral action will eliminate steady-state error (see Case 2)
- Derivative action may increase transient response but cause instability

Conclusion:

- Open-loop Step Response shows motorized winch-and-cart is a First Order System
- Resulting PID closed-loop transfer function yields Type 0
- Type 0 systems just need PI (no derivative) control. Steady-state error will be 0