ME729 Advanced Robotics - Computed-Torque Control

4/16/2018

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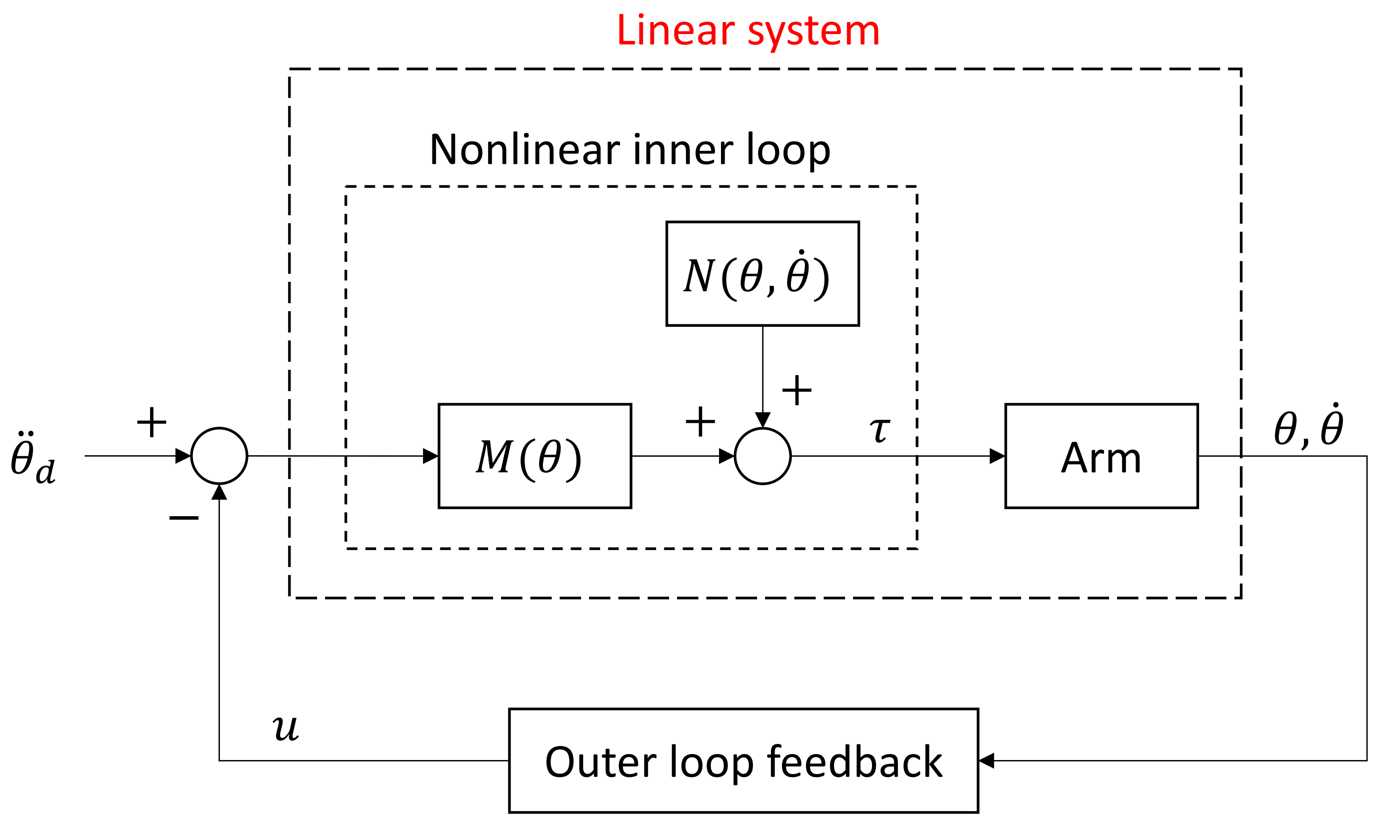
Computed-Torque Control

❑ Introduction

- A special application of feedback linearization of the nonlinear system.
- Feedforward loop : for eliminating nonlinear terms of the system.
- Feedback loop : for tracking a reference input.



Computed-Torque Control

- Derivation of inner feedforward loop
  - The robot dynamics:

$$M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) = \tau$$

  - For convenience, $V(\theta, \dot{\theta}) + G(\theta) = N(\theta, \dot{\theta})$:

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) = \tau$$

  - Define tracking error:

$$e(t) = \theta_d(t) - \theta(t)$$

where, $\theta_d(t)$ is a desired trajectory.

  - Differentiate the error twice:

$$\dot{e} = \dot{\theta}_d - \dot{\theta}, \text{and } \ddot{e} = \ddot{\theta}_d - \ddot{\theta}$$

  - To eliminate nonlinear term and track a desired trajectory, define the computed-torque control law:

$$\tau = M(\ddot{\theta}_d - u) + N$$

, where u is the outer loop feedback control input.

  - Substituting into the robot dynamics:

$$M\ddot{\theta} + N = M(\ddot{\theta}_d - u) + N$$

$$M(\ddot{\theta}_d - \ddot{\theta}) = Mu$$

$$\therefore \ddot{e} = u$$

  - **Therefore, if we select a control u that stabilizes $\ddot{e} = u$, then e goes to zero.**

Computed-Torque Control

- PD controller for the outer loop
  - One way to select the outer loop feedback control input u is the PD feedback,

$$u = -k_d\dot{e} - k_p e$$

  - Then the control law becomes

$$\tau = M\left(\ddot{\theta}_d + k_d\dot{e} + k_p e\right) + N$$

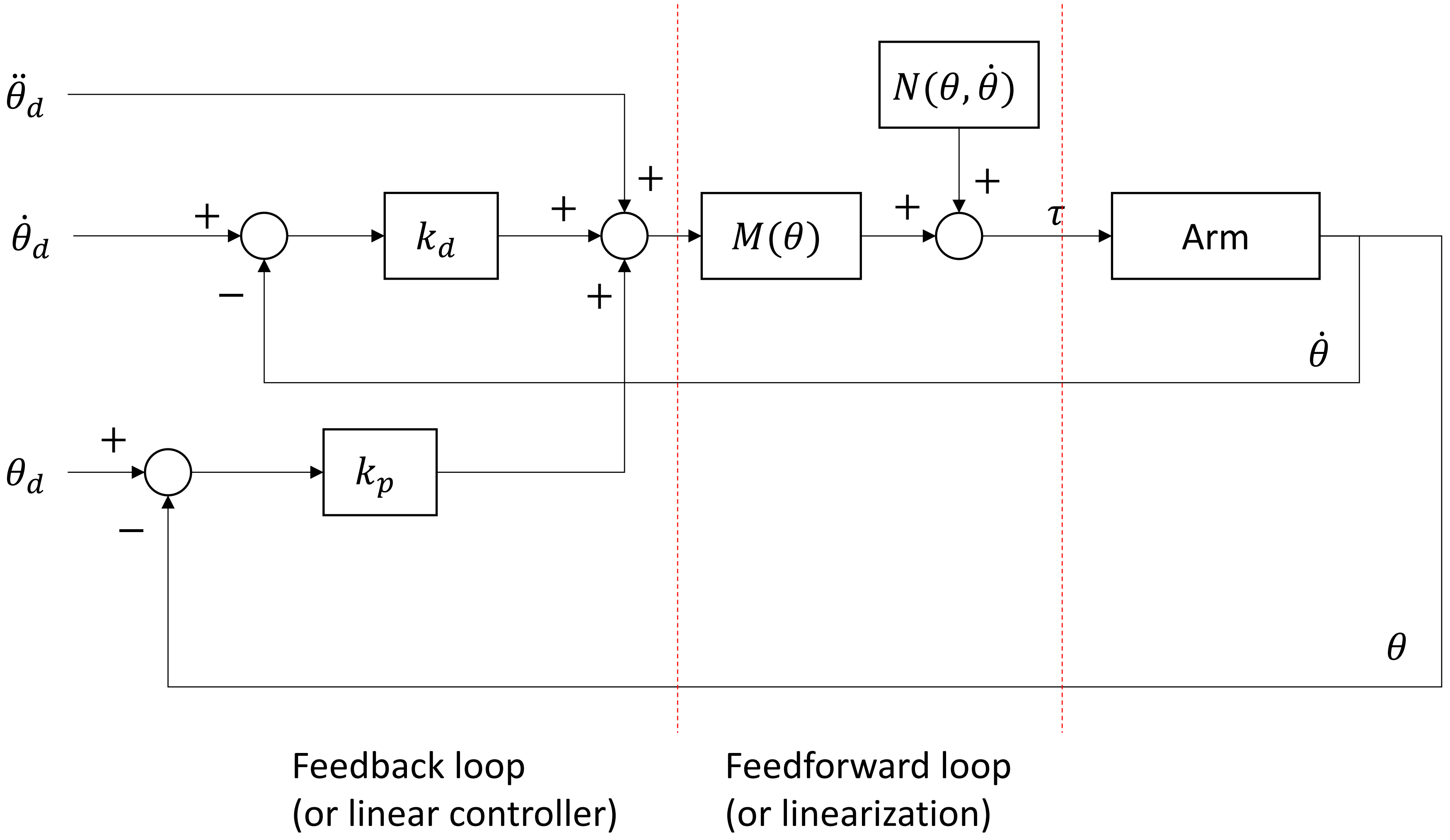
  - The closed-loop error dynamics are

$$\ddot{e} + k_d\dot{e} + k_p e = 0$$

  - The error system is asymptotically stable as long as the k_d and k_p are all positive.

Computed-Torque Control

- PD controller for the outer loop
  - Block diagram for PD computed-torque controller.



Computed-Torque Control

- PID controller for the outer loop
  - PD computed-torque control is very effective if all the arm parameters are known and there is no disturbance.
    → it gives a nonzero steady-state error.
  - Consequently, we add an integral controller in the feedback loop.

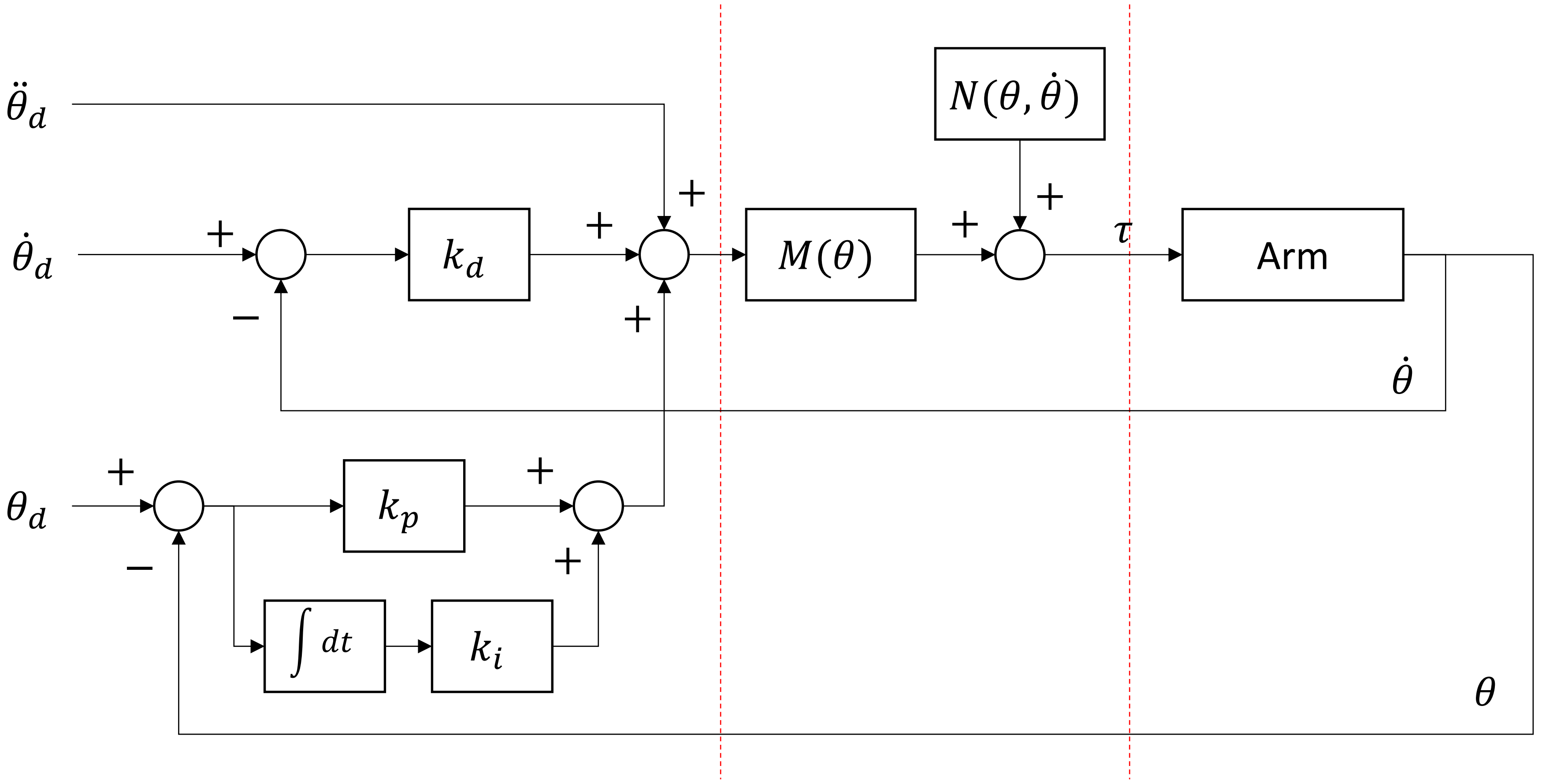
$$u = -k_d \dot{e} - k_p e - k_i \int e$$

  - Then the control law becomes

$$\tau = M\left(\ddot{\theta}_d + k_d \dot{e} + k_p e - k_i \int e\right) + N$$

Computed-Torque Control

- PID controller for the outer loop
  - Block diagram for PID computed-torque controller.

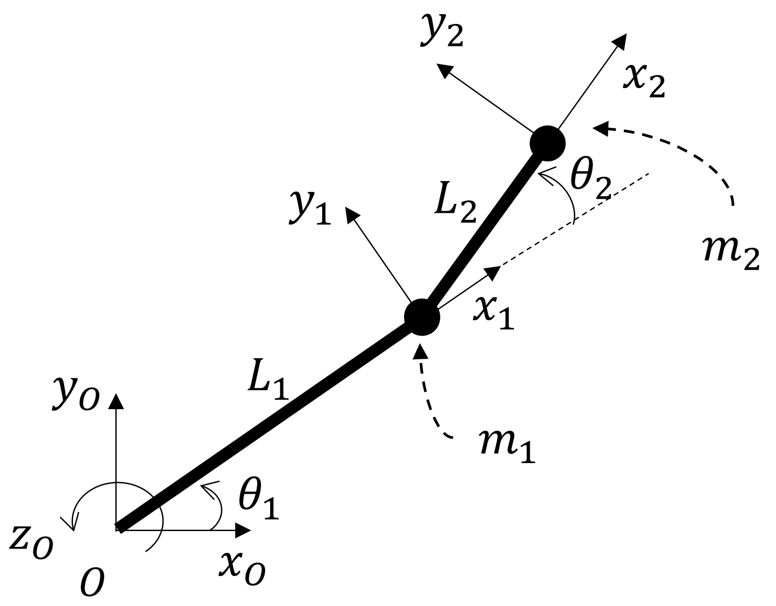


Feedback loop
(or linear controller)

Feedforward loop
(or linearization)

Computed-Torque Control

- For example
  - The two-link planar manipulator.
  - Take the link masses as 1 kg and their lengths as 1 m.



  - The manipulator's equations of motion

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta})$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 3 + 2c_2 & 1 + c_2 \\ 1 + c_2 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right) s_2 \\ \dot{\theta}_1^2 s_2 \end{bmatrix}$$

Computed-Torque Control

- For example
  - Rewrite the manipulator's equations of motion

$$\ddot{\theta} = M(\theta)^{-1}\{\tau - V(\theta, \dot{\theta})\}$$

  - The PD computed-torque control law is given as

$$\tau = M(\theta)\left(\ddot{\theta}_d + k_d\dot{e} + k_p e\right) + V(\theta, \dot{\theta})$$
$$e = \theta_d - \theta$$

  - Let the desired trajectory θ_d

$$\theta_{1d} = 0.1 \sin \pi t$$
$$\theta_{2d} = 0.1 \cos \pi t$$

  - Differentiate the desired trajectory

$$\dot{\theta}_{1d} = 0.1\pi \cos \pi t$$
$$\dot{\theta}_{2d} = -0.1\pi \sin \pi t$$
$$\ddot{\theta}_{1d} = -0.1\pi^2 \sin \pi t$$
$$\ddot{\theta}_{2d} = -0.1\pi^2 \cos \pi t$$

  - Set the gains as

$$k_p = 100, k_v = 20$$

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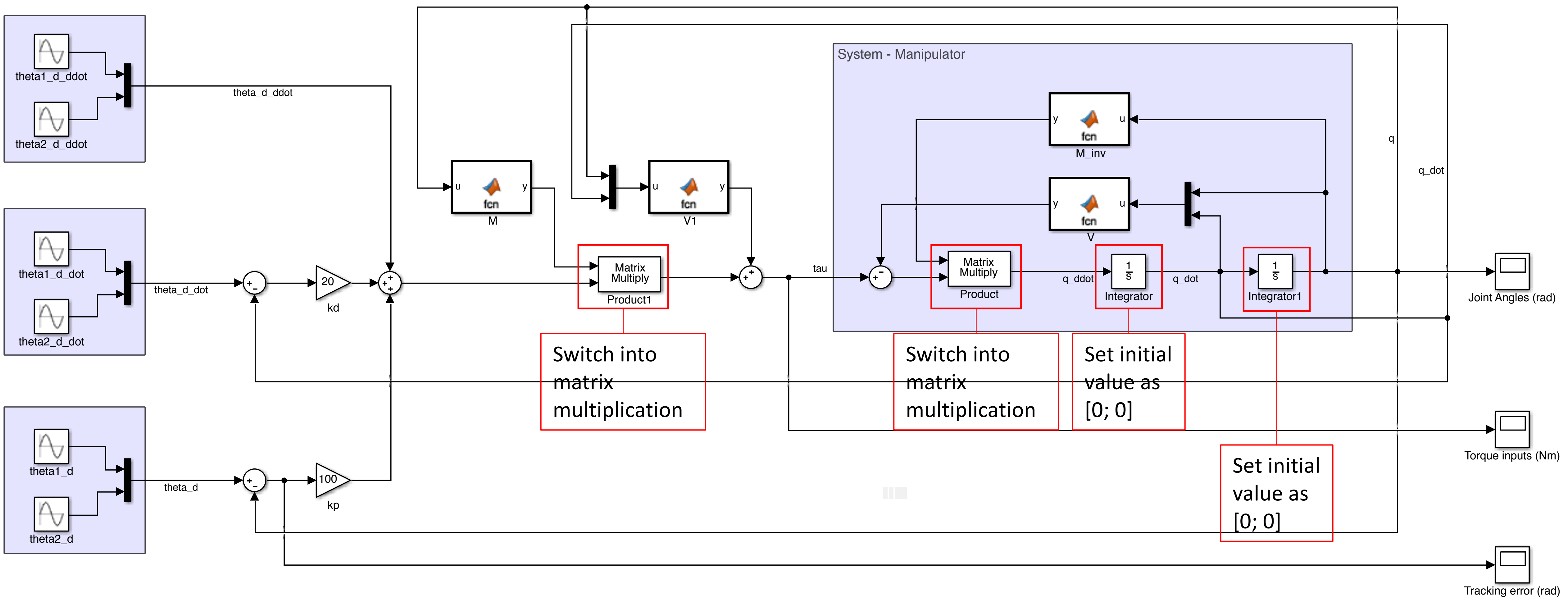
❑ For example

- Simulink blocks



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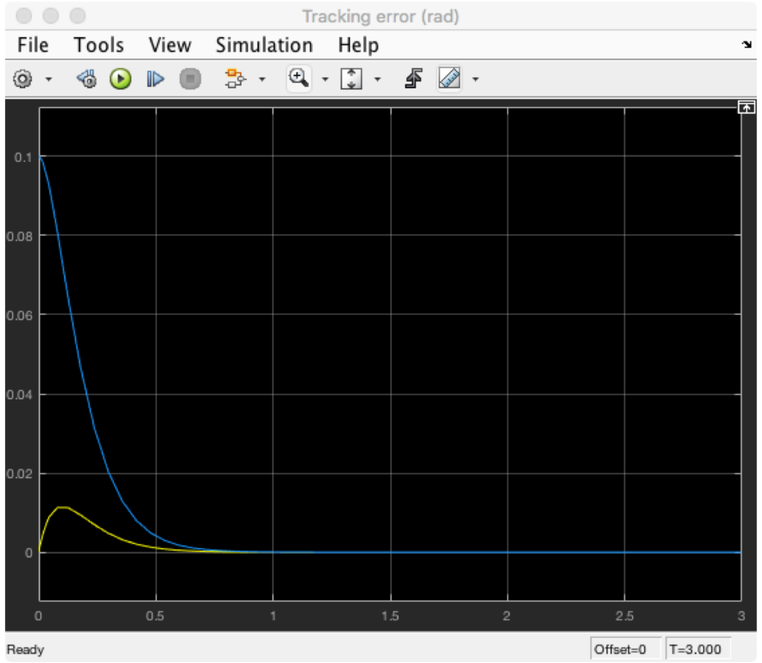
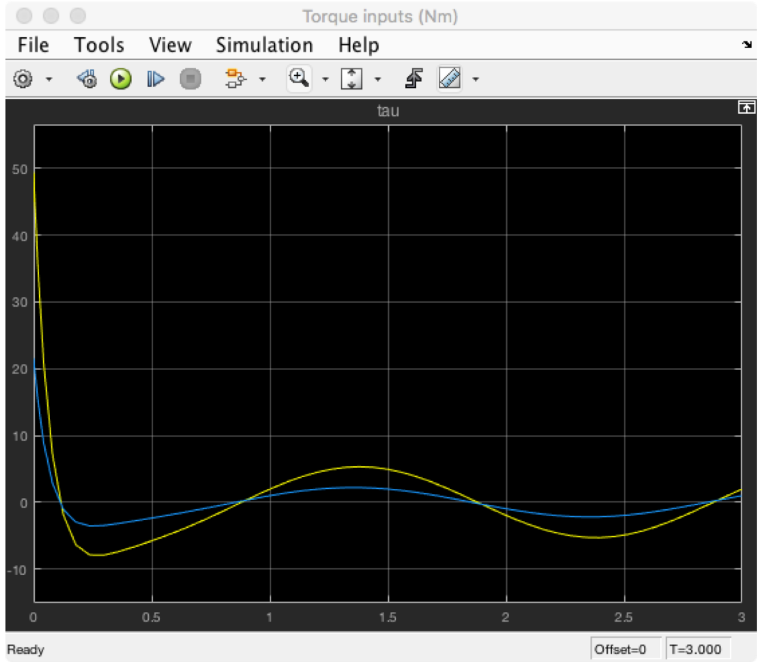
- For example
  - Simulation results
  - Yellow line is regarding to θ_1.
  - Blue line is regarding to θ_2.

[Joint angles (rad)]



[Tracking error (rad)]



[Computed torque input (Nm)]