ME729 Advanced Robotics - Force Control

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Force Control

❑ Introduction

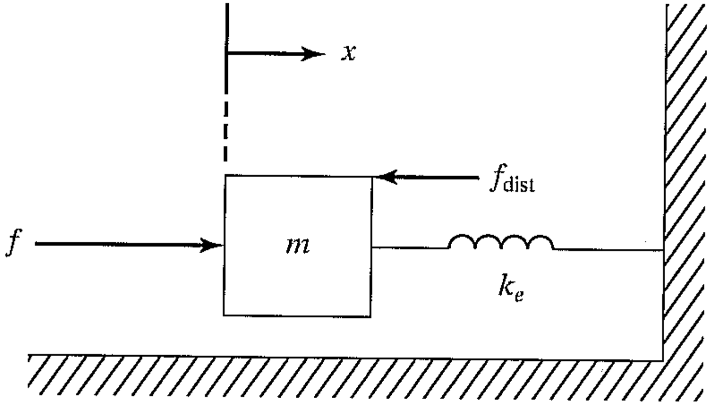
- Position control
  - when a manipulator is following a trajectory through space.

- Force control
  - when any contact made between the end-effector and the manipulator’s environment.
  - For example, a manipulator washing a window with a sponge.

Force Control

Force control of a mass-spring

- Let us consider the control of a mass attached to a spring as in a figure.



Where, f_{dist} is an unknown disturbance force, f is an input, and x is an output.

- We wish to control the force acting on the environment, f_e.
- The f_e is the force acting in the spring:

$$f_e = k_e x$$

- The equation of motion of the system is

$$f = m\ddot{x} + k_e x + f_{dist}$$

- Written in terms of f_e, we have

$$f = m k_e^{-1} \ddot{f}_e + f_e + f_{dist}$$

Force Control

- Force control of a mass-spring – continued
  - If we use the control law as

$$f = mk_e^{-1}\{\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f\} + f_e + f_{dist}$$

where, $e_f = f_d - f_e$ is the force error between the desired force, f_d, and the sensed force on the environment, f_e.

  - Substituting the control law into the equation of motion, we have the closed loop system.

$$mk_e^{-1}\{\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f\} + f_e + f_{dist} = mk_e^{-1}\ddot{f}_e + f_e + f_{dist}$$

$$\therefore \ddot{e}_f + k_{vf}\dot{e}_f + k_{pf}e_f = 0$$

  - However, we cannot use knowledge of f_{dist} in our control law, so that control law is not feasible.
  - If we choose to use f_d in the control law in place of the term $f_e + f_{dist}$,

$$f = mk_e^{-1}\{\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f\} + f_d$$

we find the steady-state error to be

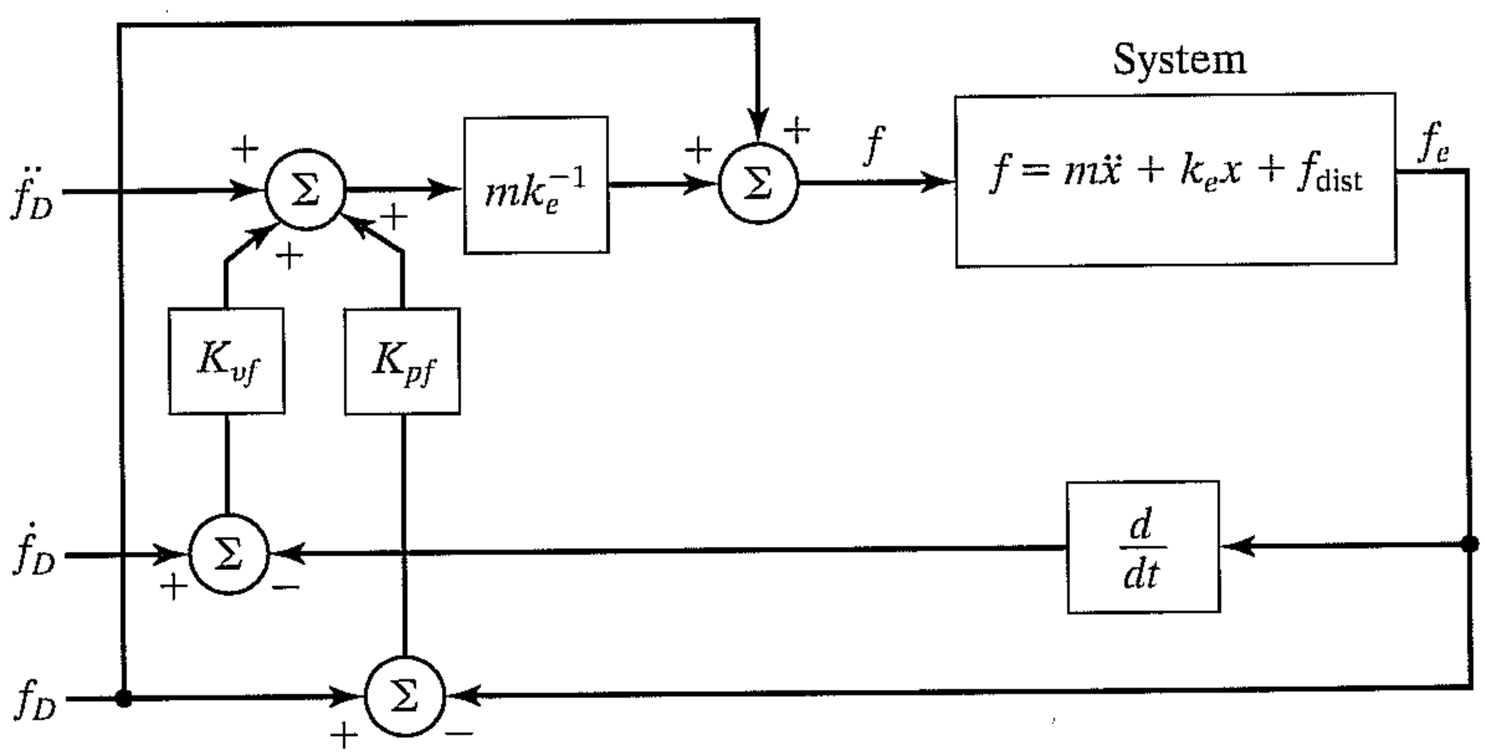
$$mk_e^{-1}\{\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f\} + f_d = mk_e^{-1}\ddot{f}_e + f_e + f_{dist}$$

$$mk_e^{-1}\{\cancel{\ddot{e}_f} + k_{vf}\cancel{\dot{e}_f} + k_{pf}e_f\} + e_f = f_{dist}$$

$$\therefore e_f = \frac{f_{dist}}{1 + \cancel{mk_e^{-1}k_{pf}}} \rightarrow 0 \because k_e \text{ is big.}$$

Force Control

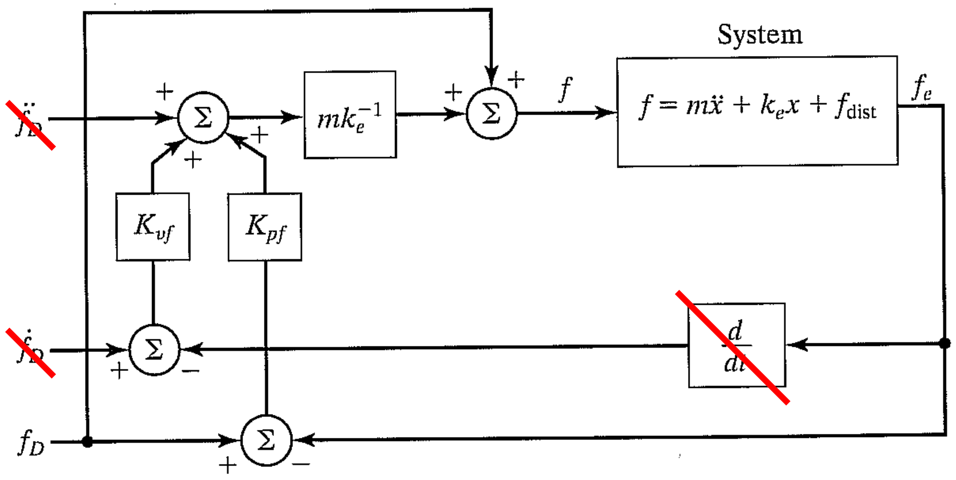
- Force control of a mass-spring – continued
  - A block diagram of the closed loop system using the control law.



Force Control

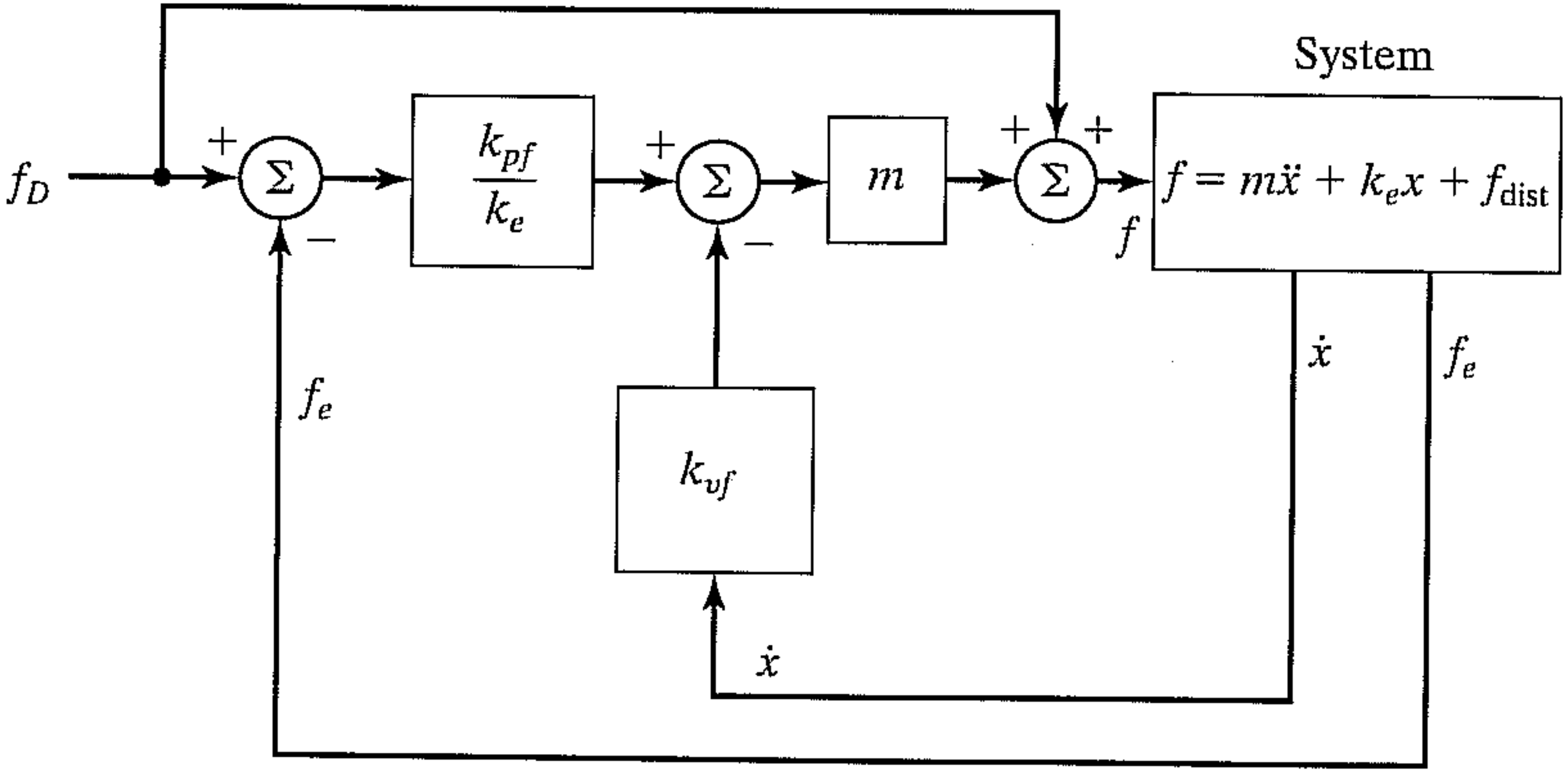
- Force control of a mass-spring – continued
  - **In practical consideration**
  - *Force trajectories are usually constants.*
    - $\dot{f}_d$ and $\ddot{f}_d$ are often set to zero.
  - *Sensed forces are quite noisy.*
    - To compute $\dot{f}_e$, use the equation, $\dot{f}_e = k_e\dot{x}$, rather than the numerical differentiation of f_e.
  - Therefore, the before control law is changed into the practical control law.

$$f = mk_e^{-1}\{\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f\} + f_d \rightarrow f = m\{k_{pf}k_e^{-1}e_f - k_{vf}\dot{x}\} + f_d$$



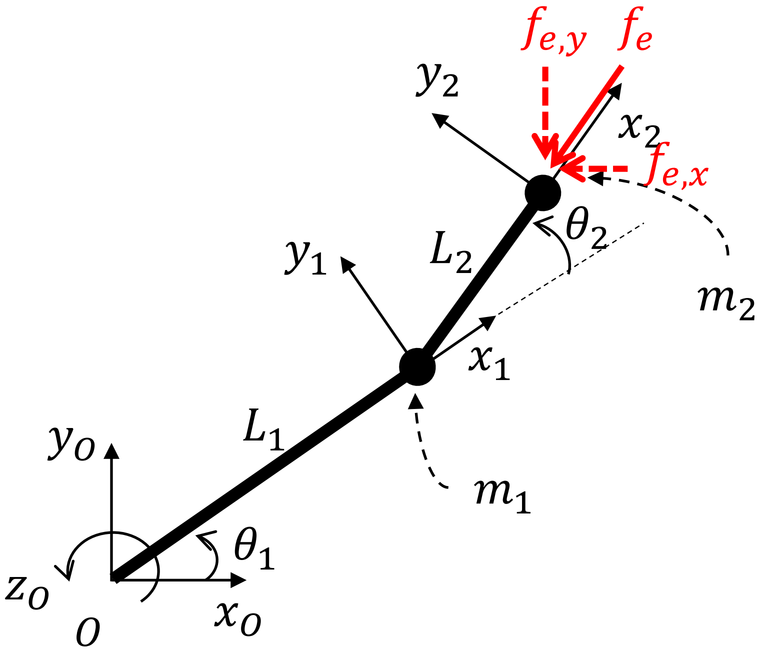
Force Control

- Force control of a mass-spring – continued
  - A practical force control system for the mass-spring.



Force Control

❑ The 2-link planar manipulator



$$f_e = \begin{bmatrix} f_{e,x} \\ f_{e,y} \end{bmatrix}$$

- The manipulator's equations of motion

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + J(\theta)^T f_e$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} (m_1 + m_2)L_1^2 + m_2 L_2^2 + 2m_2 L_1 L_2 c_2 & m_2 L_2^2 + m_2 L_1 L_2 c_2 \\ m_2 L_2^2 + m_2 L_1 L_2 c_2 & m_2 L_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -m_2 L_1 L_2 \left(2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2\right) s_2 \\ m_2 L_1 L_2 \dot{\theta}_1^2 s_2 \end{bmatrix}$$

$$+J(\theta)^T f_e$$

Force Control

- The 2-link planar manipulator
  - Desired force and error: $f_d = [f_{d,x} \quad f_{d,y}]^T$, and $e_f = [f_{d,x} - f_{e,x} \quad f_{d,y} - f_{e,y}]^T$
  - The practical control law for the manipulator

$$f = m\{k_{pf} k_e^{-1} e_f - k_{vf} \dot{x}\} + f_d$$

$$\downarrow \; \dot{x} = J(\theta)\dot{\theta}$$

$$\tau = J(\theta)^T \left[M(\theta)\{k_p e_f - k_v J(\theta)\dot{\theta}\} + f_d \right]$$

  - The block diagram of the manipulator