ME729 Advanced Robotics - PID and Linear Control

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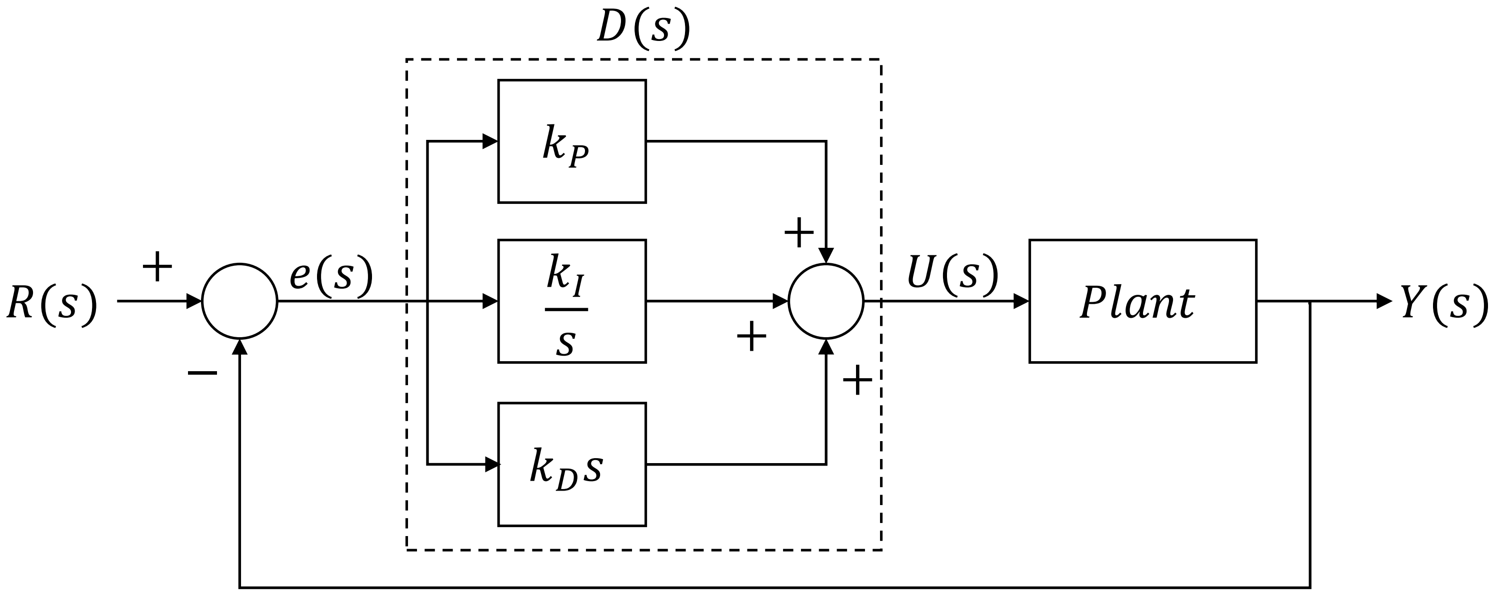
PID and Linear Control

❑ PID controller

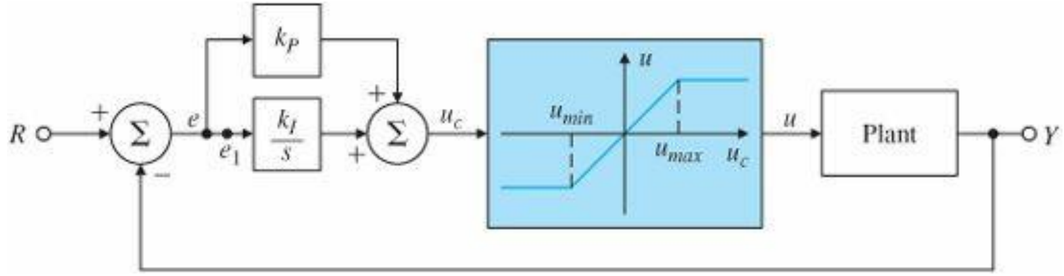
- Proportional–Integral–Derivative controller.
- $D(s) = k_P + \frac{k_I}{s} + k_D s$



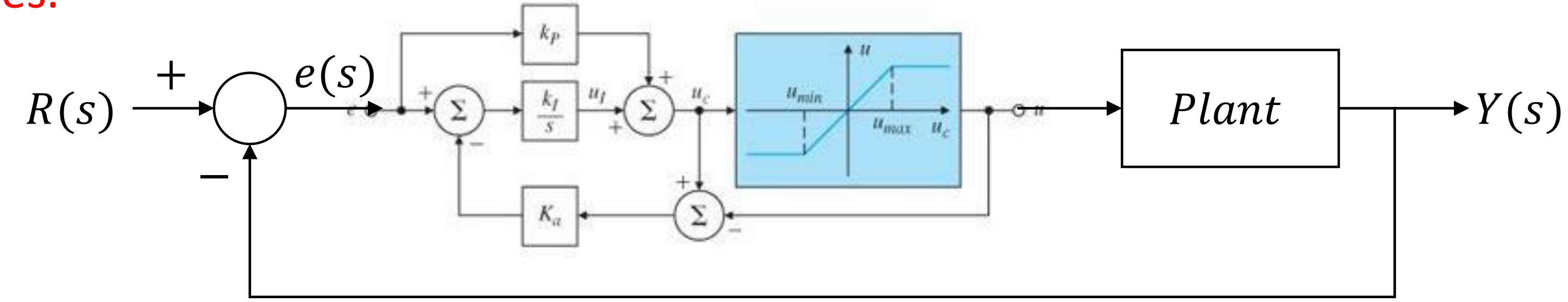
- The PID controller is broadly applicable, since it relies only on the response of the measured process variable, not on knowledge or a model of the underlying process.
- The PID gain tuning is a difficult problem.
- There are several methods for tuning the gains: manual tuning, Ziegler-Nichols method, etc.

PID and Linear Control

❑ PID controller - Integrator antiwindup



- Suppose a given reference step is more than large enough to cause the actuator to saturate at u_{max}.
- The integrator continues integrating the error e, and the signal u_c keeps growing.
- However, the input to the plant is stuck at its maximum value, namely $u = u_{max}$, so the error remains large until the plant output exceeds the reference and the error changes sign.
- The solution to this problem is an integrator antiwindup, which "turns off" the integral action when the actuator saturates.

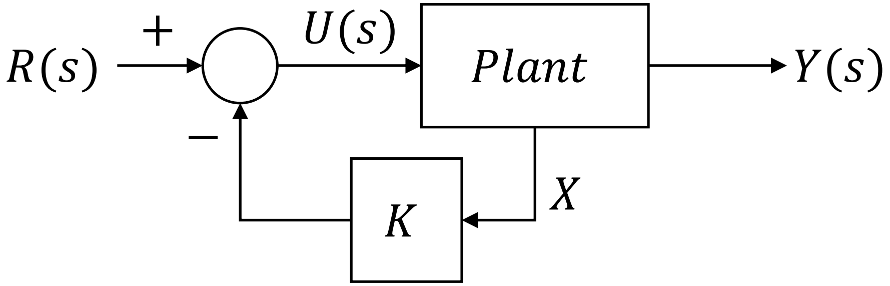


- In this scheme, as soon as the actuator saturates, the feedback loop around the integrator becomes active and acts to keep the input to the integrator at e_1 small.

PID and Linear Control

❑ Pole-placement method

- To place the closed-loop poles of a plant in pre-determined locations (desired poles) in the s-plane.



- If the closed-loop dynamics can be represented by the state space equation with output equation,

$$\dot{X} = AX + BU, \qquad Y = CX$$

then the poles of the system transfer function are the roots of the characteristic equation given by

$$\det(sI - A) = 0$$

Consider an input proportional to the state vector,

$$U = -KX$$

Substituting into the state space equations above,

$$\dot{X} = (A - BK)X$$

Therefore, the poles of the full state feedback system are given by $\det[sI - (A - BK)] = 0$.

- Determine feedback gain, K, through comparing the poles of the full state feedback system with desired poles.

PID and Linear Control

- Pole-placement method – continued
  - For example, there is a control system given by the following state space equations

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

  - Desire poles: $s = -1, and\ s = -5$
    → The desired character equation is $(s + 1)(s + 5) = s^2 + 6s + 5 = 0.$

  - Let $K = [k_1\ k_2]$,

$$|sI - (A - BK)| = \det\begin{bmatrix} s & -1 \\ 2 + k_1 & s + 3 + k_2 \end{bmatrix} = s^2 + (3 + k_2)s + (2 + k_1)$$

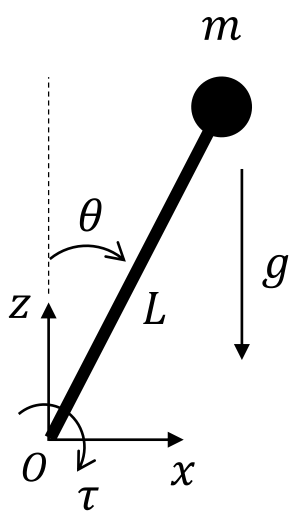
  - Comparing two equations,

$$s^2 + 6s + 5 = s^2 + (3 + k_2)s + (2 + k_1)$$

  - Therefore, $k_1 = 3, and\ k_2 = 3.$

PID and Linear Control

❑ Inverted pendulum model (IPM)



- For IPM, the kinetic and potential energies are

$$K = 1/2\, mL^2\dot{\theta}^2$$
$$P = mgL\cos\theta$$

- The Lagrangian

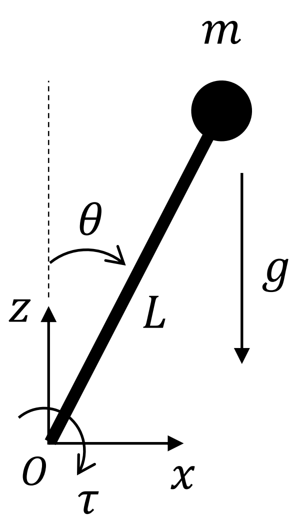
$$L = K + P = 1/2\, mL^2\dot{\theta}^2 + mgL\cos\theta$$

- Lagrangian's equation

$$\frac{\partial L}{\partial \dot{\theta}} = mL^2\dot{\theta}$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = mL^2\ddot{\theta}$$
$$\frac{\partial L}{\partial \theta} = -mgL\sin\theta$$
$$\therefore \tau = mL^2\ddot{\theta} - mgL\sin\theta$$

PID and Linear Control

❑ Inverted pendulum model (IPM) – continued



- Linearization:
  Assume that θ is small. Then, $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.
  After linearizing Lagrangian's equation,

$$\therefore \tau = mL^2\ddot{\theta} - mgL\theta$$

- State-space representation:
  Rewrite the linearized equation,

$$\ddot{\theta} = \frac{g}{L}\theta + \frac{1}{mL^2}\tau$$

Then,

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{g}{L} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{mL^2} \end{bmatrix} \tau$$

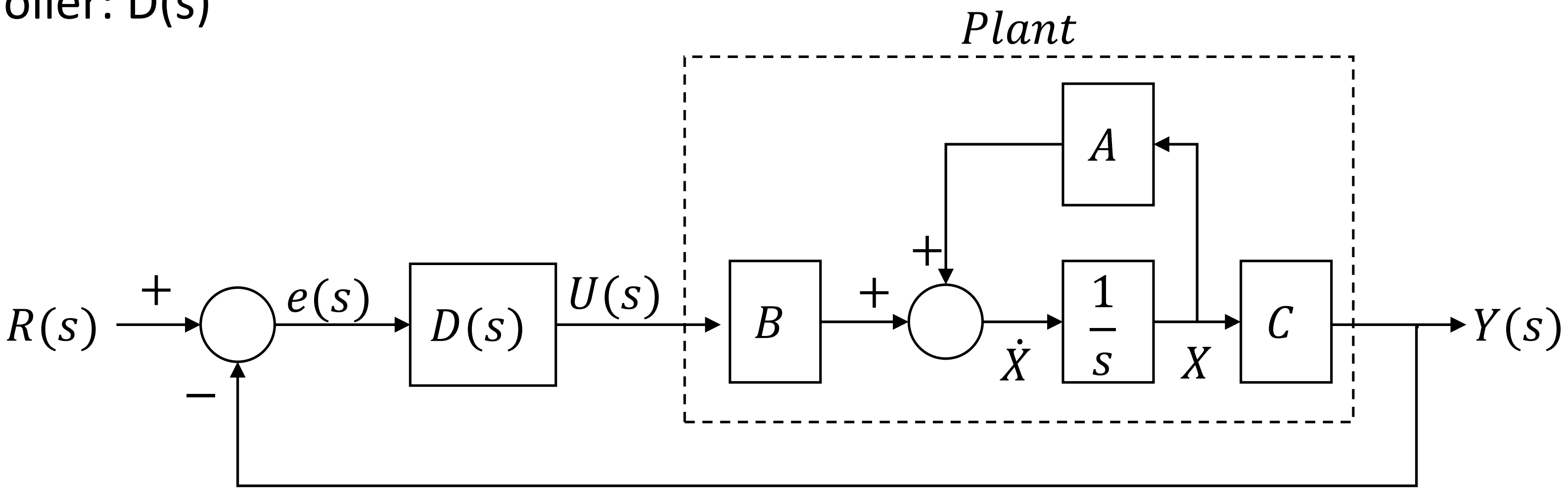
Set state as $X = [\theta \quad \dot{\theta}]^T$, input u as the joint torque τ,

$A = \begin{bmatrix} 0 & 1 \\ \frac{g}{L} & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{mL^2} \end{bmatrix}$, and $C = [1 \quad 0]$.

PID and Linear Control

❑ Inverted pendulum model (IPM) – continued

- PID controller: D(s)



- Pole-placement method