H.W. - Solution

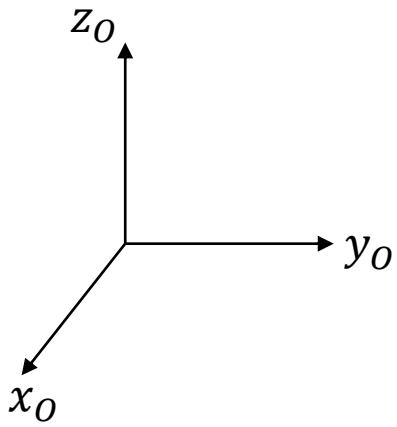
❖ **Submit *a pdf file* via email by next Monday 6 p.m.**

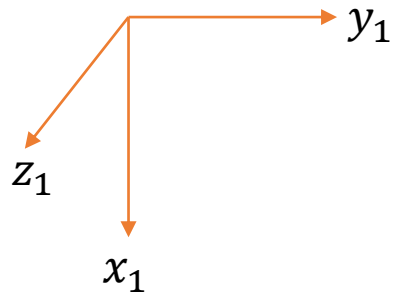
1. There is a base frame. Compute a rotation matrix. Also, sketch a final rotated frame as well as the base frame. [1.8]

   1) Rotate $-90^{\circ}, 90^{\circ}, \text{and } -90^{\circ}$ about *x*, *y*, and *z*, respectively, using Roll-Pitch-Yaw rotation.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[0.3]



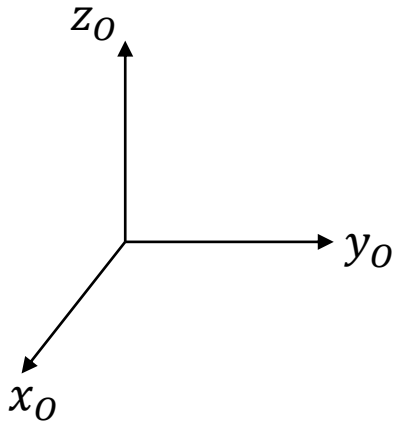


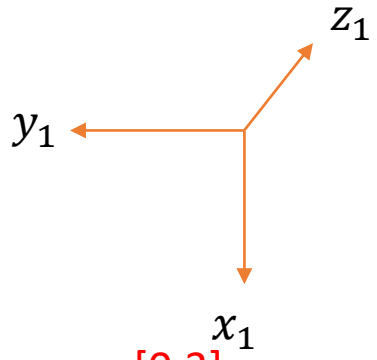
[0.3]

   2) Rotate $90^{\circ}, 90^{\circ}, \text{and } -90^{\circ}$ about *x*, *y*, and *z*, respectively, using Z-Y-X Euler angles rotation.

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[0.3]





[0.3]

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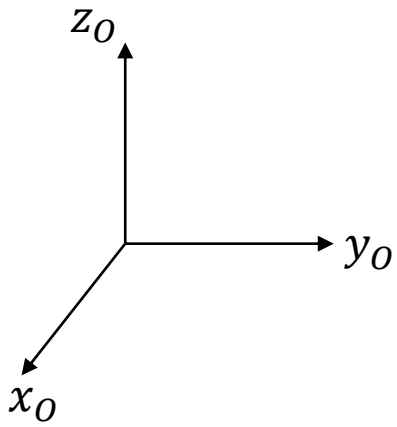
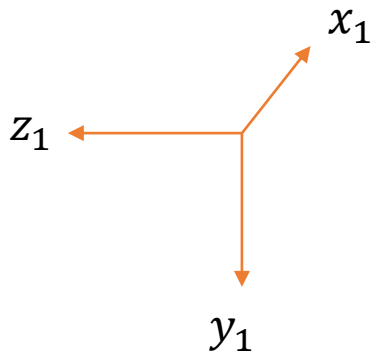
3) Rotate $90^{\circ}, -90^{\circ}, \text{and } 90^{\circ}$ about *z*, *y*, and *z*, respectively, using Z-Y-Z Euler angles rotation.

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[0.3]

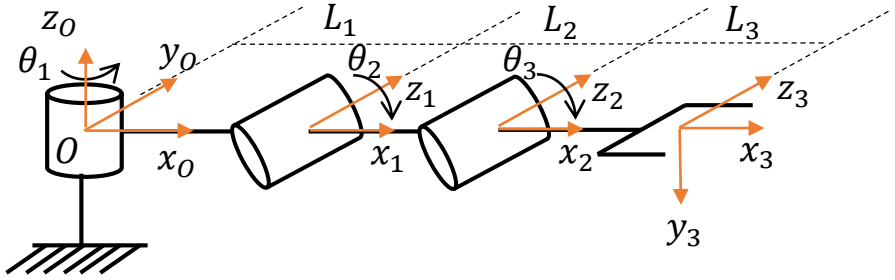




[0.3]

H.W. - Solution

2. Consider a three revolute joints (3R) manipulator. It is represented by standard DH notation. [2.1]



i	θ_i	α_i	a_i	d_i
1	θ_1	-90°	L_1	0
2	θ_2	0°	L_2	0
3	θ_3	0°	L_3	0

1) ${}^{O}A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & L_1c_1 \\ s_1 & 0 & c_1 & L_1s_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

2) ${}^{1}A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

3) ${}^{2}A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_3c_3 \\ s_3 & c_3 & 0 & L_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

4) ${}^{O}A_1\ {}^{1}A_2\ {}^{2}A_3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & -s_1 & c_1(L_1 + L_2c_2 + L_3c_{23}) \\ s_1c_{23} & c_1s_{23} & c_1 & s_1(L_1 + L_2c_2 + L_3c_{23}) \\ -s_{23} & -c_{23} & 0 & -L_2s_2 - L_3s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.6]

$*\cos(\theta_i) = c_i\ , \sin(\theta_i) = s_i, \cos(\theta_i + \theta_j) = c_{ij}\ , \sin(\theta_i + \theta_j) = s_{ij}$

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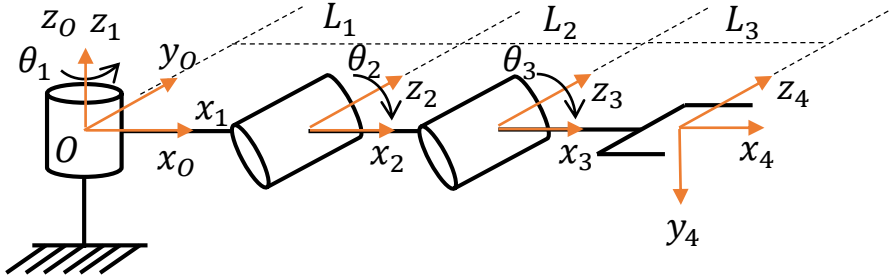
3. Consider a 3R manipulator again. It is represented by modified DH notation. [2.0]



i	θ_i	α_{i-1}	a_{i-1}	d_i
1	θ_1	0	0	0
2	θ_2	-90°	L_1	0
3	θ_3	0°	L_2	0
4	0	0°	L_3	0

1) ${}^{O}A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

2) ${}^{1}A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

3) ${}^{2}A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

4) ${}^{3}A_4 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ [0.5]

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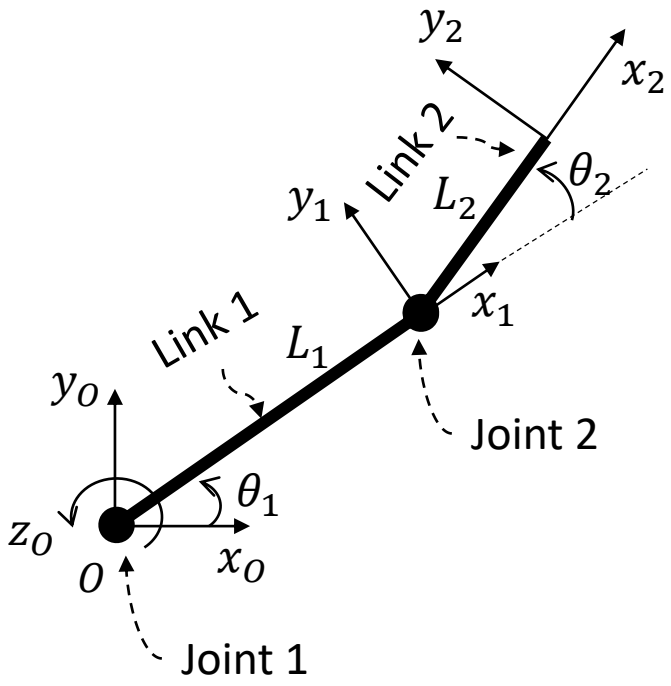
4. There is a 2R planar manipulator. The $p, L_1, and\ L_2$ are tip of the manipulator, link 1 length and link 2 length, respectively. [2.1]

1) Sketch coordinate frames and DH parameters using ***standard*** DH notation. [0.4]



2) Make a DH parameters table. [0.4]

i	θ_i	α_i	a_i	d_i	$\cos(\alpha_i)$	$\sin(\alpha_i)$
1	θ_1	0°	L_1	0	1	0
2	θ_2	0°	L_2	0	1	0

H.W. - Solution

3) What are the ***A*** matrices? [0.4]

$$ {}^{o}A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & L_1c_1 \\ s_1 & c_1 & 0 & L_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{1}A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ *\cos(\theta_i) = c_i \, , \sin(\theta_i) = s_i, \cos(\theta_i + \theta_j) = c_{ij} \, , \sin(\theta_i + \theta_j) = s_{ij} $$

4) What is the product of ***A*** matrices called ***T*** matrix? [0.5]

$$ {}^{o}T_2 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 & 0 & L_1c_1 + L_2c_1c_2 - L_2s_1s_2 \\ s_1c_2 + c_1s_2 & c_1c_2 - s_1s_2 & 0 & L_1s_1 + L_2s_1c_2 + L_2c_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & L_1c_1 + L_2c_{12} \\ s_{12} & c_{12} & 0 & L_1s_1 + L_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

5) Get the position of tip, p, using DH parameters. [0.4]

$$ p_x = L_1c_1 + L_2c_{12} $$
$$ p_y = L_1s_1 + L_2s_{12} $$

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5. Make a code which a manipulator moves as the following motion sequentially whenever an orange button is pressed. [2.0]
    1) Link 1 : 45°, and Link 2 : 0°
    2) Link 1 : 45° (keep current angle) , and Link 2 : 45°
    3) Link 1 : 135°, and Link 2 : 45°
    4) Link 1 : 180°, and Link 2 : 0°

    * **Submit a source code with a video clip link like a youtube link or cloud link, or a compressed video clip file.**